Brahmastra Academy

Celebrating Knowledge Progressively

MATHS-CDS

NUMBER SYSTEM

- **Number:** A number is a mathematical object used to count, measure and label.
- **Number System:** Number system or system of numeration is a writing system of numbers or symbols in a consistent manner. In simple words we can say number system deals with writing numbers.
- Depending on base or numbers used in the system, number system can be classified as:
  - Binary Number System
  - Octal Number System
  - Decimal Number System
  - Hexadecimal Number System
- These number systems are usually used in computer these days. But in "Mathematics" we use Decimal Number System.
- **Decimal Number System:** "Deci" means 10 thus "Decimal number system" use 0 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). We can form any number using these digits. The base of this system is also 10.
- **Notation of a Number:** Notation of a number can be done as:

  $231 = 2\times10^2 + 3\times10^1 + 1\times10^0$

  While doing notation of a number we take number from right to left for which we use two system to represent in India:
  - Indian System/ (Hindu-Arabic System)
  - International System

Indian System: We use it to read number from left to right but count it from right to left Arab, Ten Crores, Crores, Ten Lakh, Lakhs, Ten Thousand, Thousands, Hundred, Tens, Ones
10^9, 10^8, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0

International System: We use it to read number from left to right but count it from right to left Billion, Hund. M, Ten M, Million, Hund. Thou., Ten Thou., Thousands, Hundred, Tens, Ones
10^9, 10^8, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0

- While representing numbers we use two terms for them:
  - Face Value
  - Place Value
- **Face Value:** Face value is the actual value of the digit in the number.

  Ex-1: Face Value of 7 in 38786 is 7 only
- **Place Value:** Place value is the value of the digit with the position at which they occur in the number.

  Ex-2: Place Value of 7 in 38786 is 700 i.e. 7×100

Types of Numbers

- **Imaginary Number:** Imaginary Numbers are those number that are expressed as a square root of a negative number. These numbers are usually represented as "ai" where 'a' is a number and 'i' is symbol for imaginary part, where value of 'i' is $(-1)^2$. Imaginary number are usually plot on the vertical number line plane.

  Ex: 2i, 7i

- **Real Number:** Real Numbers are numbers that can be represented on number line. In simple words all numbers other than imaginary numbers are Real number.

  Ex: 1, -4, 2.3, $\frac{2}{5}$ $\sqrt{2}$ $\frac{1}{\sqrt{3}}$, ,

- **Irrational Number:** Irrational numbers are those real numbers that cannot be expressed as $"\frac{p}{q}"$ because either "p" or "q" is a non-terminating term.
  Ex: $\sqrt{2}$ $\frac{1}{\sqrt{3}}$,

  Note: Perfect square root are not irrational numbers for example, $\sqrt{4}$is not consider as irrational number since it has terminating answer, i.e., 2

- **Rational Number:** Rational numbers are those real numbers that can be expressed as $"\frac{p}{q}"$ where q is a non-zero number. Ex: 1, -4, 2.3

Integers

Integers are those numbers in which denominator is '1' or you can say numbers which are not in form of fraction or in decimal. Integers can be further classified as:

- **Positive Integers:** Integers that lie on positive side of number line.

  Ex: 1,2,3 … ∞

- **Negative Integers:** Integers that lie on negative side of number line.

  Ex: - ∞. -3, -2, -1

- **Zero:** Center of a number line which represents no value.

  Ex: 0

Fraction

Fraction means a part of the whole. Fraction usually written as $\frac{p}{q}$ where 'p' is called numerator and 'q' is called denominator. Here "p & q" both are non-zero term.

Fraction can be further classified as:

- **Proper Fraction:** Proper Fraction are those fractions where p < q.

  Ex: $\frac{2}{3}$ $\frac{5}{11}$

- **Improper Fraction:** Improper Fraction are those fractions where p > q.

  Ex: $\frac{4}{3}$ $\frac{15}{11}$,

- **Mixed Fraction:** Mixed Fraction are those fractions which can be expressed improper fraction as proper fraction and integers.

  Ex: $\frac{4}{3}$can be expressed as 1 $\frac{1}{3}$where 1 is an integer and $\frac{1}{3}$ proper fraction.

Note: We can form infinite number of fractions between any integers.

Decimal

Decimal is a fraction whose denominator is a power of 10. Ex: 1.5, 2.75, 3.873

Recurring Decimal:

- Recurring Decimal is decimal where a digit or a group of digit recur indefinitely after decimal. Ex: 0.66… …, 0.373737……
- Recurring decimal are usually known as bar.
- A bar or a line is put over a digit to show that the digit will be repeating itself indefinitely. 0.66…. = $\bar{6}$ 0.

Convert a recurring decimal into a fraction:

When we remove a decimal and bar from any digit then from the denominator we subtract 1, where denominator is written in of power of 10 and where power of 10 depends upon the digit or number of digit that are recurring

0. $\bar{6}$ $\frac{6}{10-1} = \frac{6}{9} = \frac{2}{3}$ =

0. $\overline{37}$ $\frac{37}{100-1} = \frac{37}{99}$ =

So to simply this we can say we put as many 9 in denominator as many digit are under bar.

If the decimal has a mixed recurring decimal i.e. 0.5 , t$\bar{6}$en we use zero for non-bar numbers and 9 for bar number in denominator and subtract the non-bar number from whole number in numerator. Ex: 0.5 $\bar{6}$

0.5 $\bar{6}$= $\frac{56-5}{90} = \frac{51}{90} = \frac{17}{30}$

Natural Numbers: Natural numbers can be said the number we used for counting or a set of all positive integers. Ex: 1, 2 , 3 ,….. ∞

Whole Numbers: When zero is also included in natural numbers than the set of numbers are called whole numbers. Ex:0, 1, 2 , 3 ,….. ∞

Even Numbers: Set of natural numbers which are exactly divisible by 2 is known as even number. Ex: 2, 4, 6, 8…. ∞

Odd Numbers: Set of natural number except even numbers are called odd numbers. Ex: 1, 3, 5…. ∞

Prime Numbers: Prime numbers are set of those natural numbers which have exactly two factors, those are '1' and itself. Ex: 2, 3, 5, 7, 11……

Composite Numbers: Composite numbers are set of those natural numbers which have more than two factors. Ex: 4,6,8,9,10,12…….

Unit: As we know 1 has only one factor thus it is neither prime nor composite thus classified as unit. Ex: 1

Twin Prime Numbers: Two prime numbers which differ by 2 is known as twin prime numbers. Ex: (3,5), (5,7) (11,13)

Co-prime Numbers: Co prime numbers are a set of those numbers which have no common factor between them except '1'. In other word we can say a set of numbers with H.C.F 1 are co-prime numbers. Ex: (6,35), (12,25)

Note:

- 2 is the only even prime number.
- There are 15 prime numbers between 1 and 50 and their sum is 328
- There are 25 prime numbers between 1 and 100 and their sum is 1060
- 2 and 3 are the only consecutive prime numbers.
- 3, 5 and 7 are the only triplet of twin prime numbers.

Perfect Number: A number is said to be perfect number, if the sum of their factors except that number is equal to that number. Ex: 6, 28

Factors of 6 are 1, 2, 3, 6.

1 + 2 + 3 = 6

Factors of 28 are 1, 2, 4, 7, 14, 28

1 + 2 + 4 + 7 + 14 = 28

Sum of reciprocal of the factors of a perfect number is always 2.

Ex:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=2$$

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=2$$

Divisibility Test

By 2: If the unit digit of a number is even then the number is divisible by 2 or the last digit is 0.

Ex: 47896 is divisible by 2 as 6 is an even number

47895 is not divisible by 2 as 5 is an odd number.

By 3: If the sum of the digits of a number is a multiple of 3.

Ex: 729834

7 + 2 + 9 + 8 + 3 + 4 = 33

3 + 3 =6

So, it is divisible by 3

425786

4 + 2 + 5 + 7 + 8 + 6 = 32

3 + 2 =5

Thus, it is not divisible by 3

By 4: When the number formed by the last two digit is divisible by 4 or the last two digits are 0.

Ex: 384764

64 is divisible by 4 thus the number is divisible by 4

53877

77 is not divisible by 4 thus the number is not divisible by 4

By 5: If the last digit of a number is 5 or 0 then the number is divisible by 5.

Ex: 4375815

Since last digit is 5 than the number is divisible by 5.

By 6: A number which follows divisibility test of both 2 and 3 then the number is divisible by 6.

Ex: 729834

7 + 2 + 9 + 8 + 3 + 4 = 33

3 + 3 =6

So, it is divisible by 3.
And last digit is even so it is divisible by 2 as well thus the number is divisible by 6.

By 7: Lets understand with an example.

Ex: 3402

Step 1: 340 – 2 × 2 =336

Step 2: 33 – 2 × 6 = 21

Step 3: Check 21 is divisible by 7 or not.

As it is divisible by 7 thus number is divisible by 7.

By 8: When the number formed by the last three digit is divisible by 8 or the last three digits are 0.

Ex: 847648
648 is divisible by 8 thus the number is divisible by 8

By 9: If the sum of the digits of a number is a multiple of 9

Ex: 6329834

6 + 3 + 2 + 9 + 8 + 3 + 4 = 36

3 + 6 = 9

So, it is divisible by 9 124786

1 + 2 + 4 + 7 + 8 + 6 = 28

2 + 8 = 10

Thus, it is not divisible by 9.

By 10: If the last digit of a number is 0 then the number is divisible by 10.
Ex: 375810
Since last digit is 0 than the number is divisible by 10.

By 11: Let's understand it with an example.

Ex: 273691

(2 + 3 + 9) – (7 + 6 + 1) = 0

Thus, the number is divisible by 11
If the difference between the sum of digits at even position in a number and sum of digits at odd position in a number is equal to "0 or multiple of 11", then the number is divisible by 11.

By 12: A number which follows divisibility test of both 3 and 4 then the number is divisible by 12.

Ex: 4298364

4 + 2 + 9 + 8 + 3 + 6 + 4 = 36

3 + 6 = 9

So, it is divisible by 3

And last two digit is divisible by 4 so it is divisible by 4 as well thus the number is divisible by 12.

Note:

- If a six digit number is formed by repeating a digit then the number is divisible by 3, 7, 11, 13, and 37.

  Ex: 111111, 222222, 333333

- If a six digit number is formed by repeating a two digit number then the number is divisible by 3, 7, 13, and 37.

  Ex: 272727, 353535, 565656

- If a six digit number is formed by repeating a three digit number then the number is divisible by 7, 11, and 13.

  Ex: 273273, 135135, 456456

- If the difference of a number of its thousand and remainder of its divisible by 1000 is divisible by 7 then number is divisible by 7.

  Ex: 596638

  638 – 596 = 42

  42 is divisible by 7 thus number is divisible by 7

- If the difference of a number of its thousand and remainder of its divisible by 1000 is divisible by 13 then number is divisible by 13

  Ex: 265213

  265 – 213 = 52

  52 is divisible by 13 thus number is divisible by 13

- $a^n + b^n$ is completely divisible by (a + b) if n is odd

- $a^n - b^n$ is completely divisible by (a + b) and (a – b) if n is even.

Counting Zero:

Usually in exam question are asked on counting the number of zeros at the end of a digit formed by product of numbers.

The number of zeros at the end of any number depends upon the number of power of 10 that can be formed in the number for which we have to remember that 10 is a product of 2 and 5.

$2^n \times 5^n = 10^n$

So, to find the number of zeros in any expression we have to find out the number of multiple of 2 and 5 that comes in the expression and the number of power of 10 will be equal to the power of 5 or 2 which ever power is less.

Ex: Find the number of zeros at the end of 24 × 15 × 14 × 75.

Solution:

$24 \times 15 \times 14 \times 75 = 2^3 \times 3 \times 3 \times 5 \times 2 \times 7 \times 5^2 \times 3$

$2^4 \times 5^3 \times 3^3 \times 7$

So, the minimum power between power of 2 and power of 5 is 3 so the power of 10 will be 3.

Thus, the number of zeros at the end of the number is 3.

Ex: Find the number of zeros at the end of 12 × 25 × 42 × 125.

Solution:

Power of 2 is 3, power of 5 is 5

Thus, power of 10 will be 3.

Factorial

Factorial is a product of all natural numbers from first natural number till the number.

In simple words we can say factorial of number is equal to the product of all natural number equal and less than the number.

N! = 1×2×3… (N – 1) × N.

Number of zeros in N! = $\frac{N}{5^1} + \frac{N}{5^2} + \frac{N}{5^3} + \ldots + \frac{N}{5^a}$

The process will carry on till $5^a > N$ and we will only add quotient, neglecting the remainder.

Ex: Find the number of zeros in 210!

Solution:

$\frac{210}{5} + \frac{210}{25} + \frac{210}{125}$

42+ 8 +1 = 51

OR

Step 1: $\frac{210}{5} = 42$

Step 2: $\frac{42}{5} = 8$

Step 3: $\frac{8}{5} = 1$

Number of zeros = 42 + 8 +1 = 51

Ex: Find the number of zeros in 1000!

Solution:

$\frac{1000}{5} + \frac{1000}{25} + \frac{1000}{125} + \frac{1000}{625}$

200 + 40 + 8 +1 = 249

Unit Digit

Several time in exam we have been asked about the unit digit of an expression. It is very simple to calculate unit digit of an expression if the numbers in expression are not given in a form of power.

Ex: Find the unit digit of 33 × 77 × 62 × 89 × 44 + 12834

Solution:

We will only use the unit digit of the numbers in the expression i.e. 7 × 2 × 9 × 4 + 4 = 4 + 4 = 8
We only consider unit of product also.

But it become difficult if the expression has number with power like 37^{31} as now we cannot determine the unit digit of this number. To calculate this easily we have to understand the cyclicity of unit digit with power of number.

Unit digit of every digit repeat itself and follows a cycle of 1,2 and 4.

Let's understand with an example.

If consider 2 so its cyclicity is 4, it means after every 4th power unit digit will be same.

$2^1 = 2$, $2^5 = 32$, $2^9 = 512$

$2^2 = 4$, $2^6 = 64$, $2^{10} = 1024$

$2^3 = 8$, $2^7 = 128$, $2^{11} = 2048$

$2^4 = 16$, $2^8 = 256$, $2^{12} = 4096$

From above example we can understand the cyclicity easily. As the biggest cycle is of 4 terms thus we try to remember cyclicity of 4 terms for all the digits.

Power **Number**	**1**	**2**	**3**	**4**
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1
0	0	0	0	0

Note:

From the above table we can say that,

- For 0, 1, 5, 6 their unit digits are 0, 1, 5, 6 respectively irrespective to their power.
- If the power of even number except number with unit digit zero is 4n then unit digit will be

6.

- If the power of odd number except number with unit digit 5 is 4n then unit digit will be 1.

Ex: What will be the unit digit of 37^{31}?

Remainder when power is divided by 4 i.e., $\frac{31}{4}$

Remainder will be 3

So, unit digit can be determined by 7^3 which will be 3

Ex: Find the unit digit $22^{21} \times 23^{22} \times 24^{23} \times 26^{25}$

Solution:

$22^{21} \times 23^{22} \times 24^{23} \times 26^{25}$

$2^1 \times 3^2 \times 4^3 \times 6$

$2 \times 9 \times 4 \times 6$

Unit digit will be 2.

Ex: Find the unit digit of the product of all the prime numbers less than 99

Solution:

2 × 3 × 5 × 7…… 97

Unit digit will be 0

Since product of 2 and 5 is 10, which means unit digit will be 0 for their product and anything multiplied by 0 will give zero, thus the unit digit will be 0 only.

Ex: Find the unit digit of 1! + 2! + 3! + …….. 99!.

Solution:

Unit digit From 5! will be zero, so we can say that the unit digit of this expression will be sum of unit digit of 1!, 2!, 3!, 4!

1 + 2 + 6 +4 =13

Unit digit will be 3.

Relation between Dividend, Divisor, Quotient and Remainder

Dividend = Divisor × Quotient + Remainder

Remainder is always as positive value but sometimes we also consider negative remainder to solve question fast.

How to calculate Negative remainder

Negative Remainder = Remainder – Divisor

Ex: What will be negative remainder if divide 1489 by 23.

Solution: Remainder = 17

Negative Remainder = 17 – 23 = – 6

Negative remainder is hypothetical condition used just to do fast calculation.

Ex: The divisor is 25 times the quotient and 5 times the remainder. If the remainder is 15, then find the dividend.

Solution:

Divisor = 15 × 5 = 75

Quotient = $\frac{75}{25}$ = 3

Dividend = 75 × 3 + 15 = 240

Ex: A number when divided by 56 it leaves 41 as a remainder. Find the remainder if the same number is divided by 7.

Solution:

Number = 56 × X + 41

Now if the same number is divided by 7, then

$\frac{56x + 41}{7} = \frac{56x}{7} + \frac{41}{7}$ Remainder will be 6

OR

Just check whether the previous divisor is divisible by new divisor if it is divisible then remainder will be equal to the remainder we will get after dividing the former remainder by new divisor else the answer will be cannot be determined.

In the above question,

56 is divisible by 7 thus remainder can easily be find by dividing 41 by 7.

41 divide by 7 gives 6 as remainder and that is our answer.

Ex: A number when divided by 65 it leaves 15 as a remainder. Find the remainder if the same number is divided by 15.

Solution: Since 65 is not divisible by 15.

Answer will be cannot be determined.

Ex: A number when divided by 7 it leaves 4 as a remainder. Find the remainder if the square of the same number is divided by 7.

Solution: Number = 7x + 4

Square = $(7x+4)^2 = 49x^2 + 56x + 16$

$$\frac{49x^2}{7} + \frac{56x}{7} + \frac{16}{7}$$

Remainder will be 2

OR

In such question, you can just square the former remainder and divided it with the divisor to get the required remainder.

$4^2 = 16$

16 divided by 7 leaves remainder 2.

Ex: When a number is divided by 27 it leaves 8 as the remainder. If the cube of the same number is divided by 27 then find the remainder.

Solution: $8^3 = 512$

Remainder left when 512 is divided by 27 is 26.

Thus, the required remainder will be 26.

Ex: Two numbers when divided by 18. Leaves remainder 12 and 7 respectively if the sum of these two numbers is divided by 18 then the remainder will be –

Solution:

First number = 18x + 12

Second number = 18y + 7

Sum = 18x + 12 + 18y + 7 = 18 (x + y) + 19

$$\frac{18x}{18} + \frac{19}{18}$$

Remainder will be 1

OR

Divisor = $\text{Remainder}_1 + \text{Remainder}_2 - \text{Remainder}_3$

Required Remainder = 12 + 7 – 18 = 1

This formula always works because sum of two remainders by same divisor cannot be equal or more than twice of the divisor.

Ex: When two different numbers are divided by the same number they leaves remainder 27 and 21 respectively. If the sum of both the numbers are divided by the same divisor the remainder will be 13 then find the divisor.

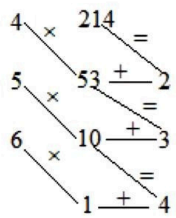
Solution: Divisor = 27 + 21 – 13 = 35

Successive Division

If in a division process quotient is used as a next dividend and the same process is carried on then the division is known as successive division. For example, if we divide 150 by 5 quotient will be 30 and remainder will be 0, now if we divide 30 by 2 quotient will be 15 and remainder will be 0, if we again divide 15 by 3 the quotient will be 5 and remainder will be 0 and now if we divide by 5 by 4 quotient will be 1 and remainder will also be 1

Ex: The least possible number when successively divided by 4, 5 and 6 leaves remainder 2, 3 and 4 respectively.

Solution:



To solve such question we consider the last quotient to be 1 and solve the question in reverse of the order i.e. we will find second last quotient first then third last and in final the smallest dividend or the number from which the successive division is started.

Second last quotient = 6 × 1 + 4 = 10

Third last quotient = 5 × 10 + 3 = 53

Smallest dividend = 4 × 53 + 2 = 214

Or {(6 + 4) × 5 + 3} × 4 + 2 =214

Ex: A least number when successively divided by 2, 3 and 5 it leaves remainder 1, 2 and 4 respectively. Find the remainder if the same number is divided by 6.

Solution: Number = {(2 + 1) × 3 + 2} × 5 + 4 = 44

If we divide 44 by 6 then remainder will be 2

Note:

- If $(1 + a)^n$ is divided by "a" then the remainder will be 1. It can also be expressed as if a^n is divided by (a – 1) the remainder will be 1.
- If a^n is divided by (1 + a) gives remainder 1 when n is even.

- If a^n is divided by (1 + a) gives remainder a when n is odd.
- If a^n + a is completely divisible by (1 + a) when n is even.
- If a^n + a is divided by (1 + a) gives remainder a – 1 when n is odd.

Practice Questions:

Q1. If $49^{15} - 1$ is exactly divisible by:

1) 5 2) 6 3) 7 4) 9

Q2. Find the remainder when $29^{47} + 17^{47}$ is divided by 46.

1) 0 2) 1 3) 7 4) 13

Q3. Find the unit digit of $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234$.

1) 6 2) 7 3) 0 4) 4

Q4. Find the unit digit of $278^{9235!} + 222^{9235!} + 666^{9235!}$

1) 6 2) 2 3) 8 4) 0

Q5. Find the number of zeroes at the end of the product 25! × 32! ×45!.

1) 10 2) 23 3) 22 4) 7

Q6. Find the number of zeroes at the end of 41×42×43…..100

1) 14 2) 15 3) 16 4) 17

Q7. If 5724A is divisible by 11 then find the value of A.

1) 0 2) 1 3) 2 4) 4

Q8. When a natural number N is divided by 3, the remainder will be 1 and when N + 1 is divided by 5, the remainder will be 0. The value of N will be:

1) 65 2) 64 3) 63 4) 62

Q9. A number when divided by 136, leaves 46 as the remainder. If the same number is divided by 34 then the remainder will be:

1) 2 2) 6 3) 12 4) 16

Q10. Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

1) 425 2) 475 3) 495 4) 535

Solution:

1) $49^{15} - 1 = 7^{2\times15} - 1^{30}$

$a^n - b^n$ is always divisible by a + b and a – b if n is even, so

$7^{30} - 1^{30}$ is always divisible by 7 + 1 = 8, 7 – 1 = 6

Thus, answer will be 6.

2) $29^{47} + 17^{47}$

$a^n + b^n$ is completely divisible by a + b when n is odd

so, 29 + 17 = 46

Thus, remainder will be 0.

3) $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234$

$3^{4n} \times 8^1 + 4^1 \times 7^{4n} + 4$

$1\times 8 + 4\times1 + 4 = 16$

Thus unit digit will be 6.

4) $278^{9235!} + 222^{9235!} + 666^{9235!}$

9235! Last 2 digit will be 0 it means all the power are in the form of 4n and all the numbers are even so the unit digit in all the case will be 6.

6 + 6 + 6 = 18

Thus, unit digit will be 8.

5) 25! × 32! × 45!

Number of zeros at the end of

25! ⇒ 5 + 1 = 6

32! ⇒ 6 + 1 = 7

45! $\Rightarrow$ 9 + 1 = 10

So total number of zeros = 6 + 7 + 10 = 23

6) Number of zeros in 40! = 8 +1 = 9

   Number of zeros in 100! = 20 + 4 = 24

   Number of Zeros in 41×42×43…..100 = 24 – 9 = 15

7) If 5724A is divisible by 11, it means

   (5 + 2 + A) – (7 + 4) = 0 or 11

   A = 4

8) The condition will be fulfilled with multiple of 4 but not a multiple of 3 and 5. So, by option we ca say 64 is the answer.

9) By dividing 46 by 34 we will get remainder i.e., 12.

10) 473 + 298 – 236 = 535

ANSWER KEY				
1) 2	2) 1	3) 1	4) 3	5) 2
6) 2	7) 4	8) 2	9) 3	10) 4

TIME, DISTANCE AND SPEED

Definition

Speed:	It is defined as the rate of travel to cover a certain distance. It is generally expressed in m/s, km/hr etc.
Time:	It is defined as the duration for which travelling has been done to cover a certain distance. It is generally expressed in seconds, hours etc.
Distance:	It is defined as the length of path for which travelling has been done. It is generally expressed in metre, kilometre etc.

Speed = $\frac{Distance}{Time}$

Unit Conversions

1) km/hr to m/s

X km/hr = (X ×$\frac{5}{18}$) m/sec

Q-1. Convert 54 km/hr into

m/sec.

Solution:

54 km/hr=54 × 5/18 =15 m/sec

2) m/s to km/hr

X m/Sec = (X × $\frac{18}{5}$) Km/hr

Q2. A car goes 20 meters in a second. Find its speed in km/hr.

Solution:
20 m/sec = 20 × 18/5 = 72 km/hr.

Ratios of Speed, Distance and/or Time

If the ratio of the speeds of A and B is a : b, then the ratio of the times take by them to cover the same distance will be $\frac{1}{a}$: $\frac{1}{b}$ or b: a.

Q-3. The speed of three cars is in the ratio 5: 4: 6. The ratio between the time taken by them to travel the same distance is

Solution:

Ratio of time taken = $\frac{1}{5}$: $\frac{1}{4}$: $\frac{1}{6}$ = 12: 15: 10

Average Speed

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total Time}}$$

Q4. A truck covers a distance of 1200 km in 40 hours. What is the average speed of the truck?

Solution:

Average speed = Total distance travelled/Total time taken

⇒ Average speed = 1200/40

∴ Average speed = 30 km/hr.

Q5. A man travelled 12 km at a speed of 4 km/h and further 10 km at a speed of 5 km/hr. What was his average speed?

Solution:

Total time taken = Time taken at a speed of 4 km/h + Time taken at a speed of 5 km/ h

⇒ 12/4 + 10/5 = 5 hours [∵ Time = Distance/Speed] Average

speed = Total distance/Total time

⇒ (12 + 10) /5 = 22/5 = 4.4 km/h

Q6. Rahul goes Delhi to Pune at a speed of 50 km/h and comes back at a speed of 75 km/h. Find his average speed of the journey.

Solution:

As, distance is same both cases

⇒ Required average speed = (2 × 50 × 75)/ (50 + 75) = 7500/125 = 60 km/hr.

Practice Questions:

Q1.The speeds of the Shaan and Rohan are 50 km/h and 30 km/h respectively. Initially Shaan is at a place N and Rohan is at a place M. The distance between M and N is 710 km. Shaan started his journey 3 hours earlier than Rohan to meet each other. If they meet each other at a place R somewhere between M and N. then the distance between R and N is

A) 210 km

B) 500 km

C) 430 km

D) 620 km

E) None of these

Ans: (B) 500km

Q2. The distance between two places A and B is 370 km. The 1st car departs from place A to B, at a speed of 80 kmph at 10 am and the 2nd car departs from place B to A at a speed of 50 kmph at 1 pm. At what time both cars meet each other?

A) 2: 30 pm

B) 2: 00 pm

C) 2: 10 pm

D) 2: 20 pm

E) None of these

Ans: (B) 2:00pm

Q3. A man takes 5 hours 45 minutes to walk to a certain place and ride back. He would have saved 2 hours had he ridden both ways. The time he would take to walk both ways is

A) 3 hours 45 minutes

B) 7 hours 30 minutes

C) 7 hours 45 minutes

D) 11 hours 45 minutes

E) None of these

Ans: (C) 7 hours 45 minutes

Q4. A and B start at the same time with speeds of 40 km/hr and 50 km/hr respectively. If in covering the journey A takes 15 minutes longer than B, the total distance of the journey is

A) 46 km

B) 48 km

C) 50 km

D) 52 km

E) None of these

Ans: (C) 50km

Q5. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km /hr of the cyclist?

A) 12 km/hr

B) 15 km/hr

C) 18 km / hr

D) 20 km / hr

Ans: (C) 18km/hr

Q6. A Jackal takes 4 leaps for every 5 leaps of goat but 3 leaps of a Jackal are equal to 4 leaps of the goat. compare their speeds

A) 12: 10

B) 7: 5

C) 1: 4

D) 16: 15

Ans: (D) 16:15

L.C.M AND H.C.F

The full forms of H.C.F. and L.C.M. are, Highest Common factor and Least Common Multiple, respectively. The H.C.F. defines the greatest factor present in between given two or more numbers, whereas L.C.M. defines the least number which is exactly divisible by two or more numbers. H.C.F. is also called the greatest common factor (GCF) and LCM is also called the Least Common Divisor.

HCF (Highest Common Factor)

As the rules of mathematics dictate, the greatest common divisor or the gcd of two or more positive integers happens to be the largest positive integer that divides the numbers without leaving a remainder. For example, take 8 and 12. The H.C.F. of 8 and 12 will be 4 because the highest number that can divide both 8 and 12 is 4.

LCM (Least Common Multiple)

In arithmetic, the least common multiple or LCM of two numbers say a and b, is denoted as LCM (a,b). And the LCM is the smallest or least positive integer that is divisible by both a and b. For example, let us take two positive integers 4 and 6.

Multiples of 4 are: 4,8,12,16,20,24…

Multiples of 6 are: 6,12,18,24….

The common multiples for 4 and 6 are 12,24,36,48…and so on. The least common multiple in that lot would be 12. Let us now try to find out the LCM of 24 and 15.

2	24, 15
2	12, 15
2	6, 15
3	3, 15
5	1, 5
	1, 1

LCM of 24 and 15 = 2 × 2 × 2 × 3 × 5 = 120

LCM of Two Numbers

Suppose there are two numbers, 8 and 12, whose LCM we need to find. Let us write the multiples of these two numbers.

8 = 16, 24, 32, 40, 48, 56, …

12 = 24, 36, 48, 60, 72, 84,…

You can see, the least common multiple or the smallest common multiple of two numbers, 8 and 12 is 24.

HCF and LCM Formula

The formula which involves both HCF and LCM is:

Product of Two numbers = (HCF of the two numbers) x (LCM of the two numbers)

Say, A and B are the two numbers, then as per the formula;

A x B = H.C.F.(A,B) x L.C.M.(A,B)

We can also write the above formula in terms of HCF and LCM, such as:

H.C.F. of Two numbers = Product of Two numbers/L.C.M of two numbers

L.C.M of two numbers = Product of Two numbers/H.C.F. of Two numbers

HCF and LCM Relation

The followings are the relation between HCF and LCM. Go through the relation between HCF and LCM, solve the problem using the relations in an easy way.

(i) The product of LCM and HCF of the given natural numbers is equivalent to the product of the given numbers.

From the given property, LCM × HCF of a number = Product of the Numbers

Consider two numbers A and B, then.

Therefore, LCM (A , B) × HCF (A , B) = A × B

Example: Show that the LCM (6, 15) × HCF (6, 15) = Product (6, 15)

Solution: LCM and HCF of 6 and 15:

6 = 2 × 3

15 = 3 x 5

LCM of 6 and 15 = 30

HCF of 6 and 15 = 3

LCM (6, 15) × HCF (6, 15) = 30 × 3 = 90

Product of 6 and 15 = 6 × 15 = 90

Hence, LCM (6, 15) × HCF (6, 15)=Product(6, 15) = 90

(ii) The LCM of given co-prime numbers is equal to the product of the numbers since the HCF of co-prime numbers is 1.

So, LCM of Co-prime Numbers = Product Of The Numbers

Example: 17 and 23 are two co-prime numbers. By using the given numbers verify that, LCM of given co-prime Numbers = Product of the given Numbers

Solution: LCM and HCF of 17 and 23:

17 = 1 x 7

23 = 1 x 23

LCM of 17 and 23 = 391

HCF of 17 and 23 = 1

Product of 17 and 23 = 17 × 23 = 391

Hence, LCM of co-prime numbers = Product of the numbers

(iii) H.C.F. and L.C.M. of Fractions

LCM of fractions = LCM of Numerators / HCF of Denominators

HCF of fractions = HCF of Numerators / LCM of Denominators

Example: Find the LCM of the fractions 1/2 , 3/8, 3/4
Solution:

LCM of fractions = LCM of Numerators/HCF of Denominators

LCM of fractions = LCM (1,3,3)/HCF(2,8,4) = 3/2

Example: Find the HCF of the fractions 3/5, 6/11, 9/20

HCF of fractions HCF of Numerators/LCM of Denominators

HCF of fractions = HCF (3,6,9)/LCM (5,11,20) = 3/220

How to Find LCM and HCF?

We can find HCF and LCM of given natural numbers by two methods i.e., by prime factorization method or division method. In the **prime factorization method**, given numbers are written as the product of prime factors. While in the division method, given numbers are divided by the least common factor and continue still remainder is zero.

Note: *Prime numbers are numbers which have only two factors i.e. one and the number itself.*

LCM by Prime Factorization Method

Here, given natural numbers are written as the product of prime factors. The lowest common multiple will be the product of all prime factors with the highest degree (power).

Example: Find the LCM of 20 and 12 by prime factorization method.

Solution:

Step 1: To find LCM of 20 and 12, write each number as a product of prime factors.

20= 2 × 2 × 5 = 22 × 5

12 = 2 × 2 × 3 = 22 × 3

Step 2: Multiply all the prime factors with the highest degree.

Here we have 2 with highest power 2 and other prime factors 3 and 5. Multiply all these to get LCM.

LCM of 20 and 12 = 2 × 2 × 3 × 5 = 2^2 × 3 × 5 = 60

LCM by Division Method

In this method, divide the given numbers by common prime number until the remainder is a prime number or one. LCM will be the product obtained by multiplying all divisors and remaining prime numbers.

Example: Find the LCM of 24 and 15 by the division method.

Solution:

Step 1: Divide the given numbers by the least prime number.

Here, 2 is the least number which will divide 24.

2	24, 15

Step 2: Write the quotient and the number which is not divisible by the above prime number in the second row.

In the second row, write the quotient we get after the division of 24 by 2. Since 15 is not divisible by 2, write 15 in the second row as it is.

Step 3: Divide the numbers with another least prime number.

2	24, 15
2	12 , 15

Step 4: Continue division until the remainder is a prime number or 1.

2	24, 15
2	12 , 15
2	6 , 15
3	3 , 15
5	1 , 5
	1 , 1

Step 5: Multiply all the divisors and remaining prime number (if any) to obtain the LCM.

LCM of 24 and 15= $2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5 = 120$

HCF By Prime Factorization Method

Given natural numbers to be written as the product of prime factors. To obtain the highest common factor multiply all the common prime factors with the lowest degree (power).

Example: Find the HCF of 20 and 12 by prime factorization method.

Solution:

Step 1: To find HCF of 20 and 12, write each number as a product of prime factors.

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

Step 2: Multiply all the common prime factors with the lowest degree.

Here we have only 2 as a common prime factor with the lowest power of 2.

HCF of 20 and 12 = $2^2 = 4$

HCF by Division Method

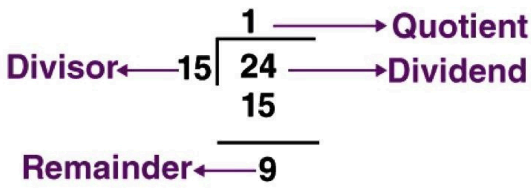
In this method divide the largest number by the smallest number among the given numbers until the remainder is zero. The last divisor will be the HCF of given numbers.

Example: Find the LCM of 24 and 15 by the division method.

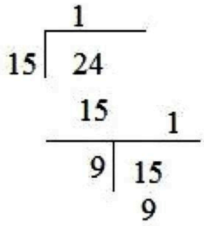
Solution:

Step 1: Divide the largest number by the smallest number.

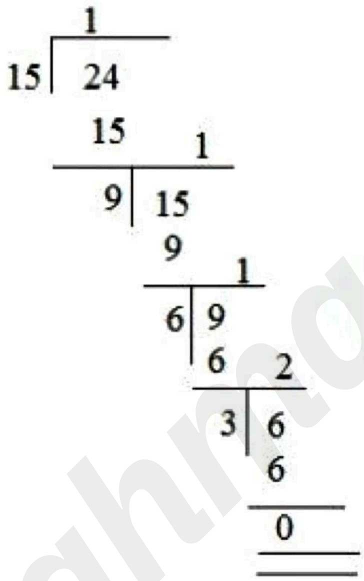
Here, the largest number is 24 and the smaller one is 15. Divide 24 by 15



Step 2: Take divisor as new dividend and remainder as the new divisor, i.e. divide the first divisor by the first remainder.



Step 3: Proceed till the remainder is zero and the last divisor will be the HCF of the given numbers.



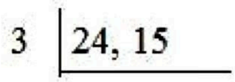
Therefore, HCF of 24 and 15 is 3.

Alternatively, we can divide both the numbers by the least common prime factor, still there is no more common prime factors. Multiply all divisors to get the HCF of given numbers.

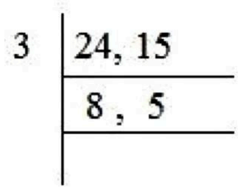
Consider the above example, HCF of 24 and 15 can also be calculated using the following steps:

Step 1: Divide the given numbers by the least common prime factor.

Here, 3 is the least common prime factor of 24 and 15.



Step 2: Continue still there is no more common prime factor. Then multiply all the divisors.



Division of 24 and 15 by 3 will leave 8 and 5 as their remainders respectively. 8 and 5 do not have a common prime factor.

Hence, the HCF of 24 and 15 is 3.

Solved Examples

Example 1: Find the Highest Common Factor of 25, 35 and 45.

Solution: Given, three numbers as 25, 35 and 45.

We know, 25 = 5 × 5

35 = 5 × 7

45 = 5 × 9

From the above expression, we can say 5 is the only common factor for all the three numbers.

Therefore, 5 is the HCF of 25, 35 and 45.

Example 2: Find the Least Common Multiple of 36 and 44.

Solution: Given, two numbers 36 and 44. Let us find out the LCM, by division method.

	36, 44
2	18, 22
2	9, 11
3	3, 11
3	1, 11
11	1, 1

Therefore, LCM (36, 44) = 2 × 2 × 3 × 3 × 11 = 396

Example 3: What is the L.C.M. of 25, 30, 35 and 40?

Solution: L.C.M. of 25, 30, 35 and 40

Let us find LCM by prime factorisation.

Prime factorisation of 25 = 5 x 5 = 5^2

Prime factorisation of 30 = 2 x 3 x 5

Prime factorisation of 35 = 5 x 7

Prime factorisation of 40 = 2 x 2 x 2 x 5 = 2^3 x 5

Thus, LCM (25, 30, 35, 40) = 2 x 2 x 2 x 3 x 5 x 5 x 7 = 4200

Example 4: The HCF of two numbers is 29 & their sum is 174. What are the numbers?

Solution: Let the two numbers be 29x and 29y.

Given, 29x + 29y = 174

29(x + y) = 174

x + y = 174/29 = 6

Since x and y are co-primes, therefore, possible combinations would be (1,5), (2,4), (3,3)

For (1,5): 29 x = 29 x 1 and 29 y = 29 (5) = 145

Therefore, the required numbers are 29 and 145.

DATA INTERPRETATION (DI)

Data interpretation refers to the process of reviewing provided data and to use these data for calculating the required value. The data can be provided in various forms like in table format, pie chart, line graph, bar graph, caselet or a combination of these.

Points to Remember

- **Read the entire question carefully** – Read the complete data given in the form of values, graph etc.
- **Analyze the data** – Take a look and analyze the data carefully. Don't get diverted or afraid due to a lot of information and avoid skipping the information before giving a glance to it.
- **Pay attention to the units** – Many times, different units are used in one question. For example, speed is given in km/h and time is to be calculated in seconds.
- **Use of approximation** – If the options are adequately far apart then you can approximate values, fractions and percentages to nearby numbers which can ease our calculations.
- **Use of last Digit** – Check if all options have different last digits, then to find the correct option, we can just calculate the last digit of our answer (but then approximation is not at all allowed).
- **Mental calculations** – Try to do mental calculations as frequently as possible while practicing. It will help in minimizing the time to solve the question.

Types of Data Interpretation

- Tabular DI
- Pie chart
- Bar graph
- Line graph
- Caselet DI

Tabular DI

In this data is provided in horizontal rows and vertical columns called tabular form. We need to understand the given information and thereafter answer the given questions.

Directions: **Study the following information carefully and answer the given questions based on it.**

Table shows the number of trees planted by the government in 6 different years.

	Banyan	Neem	Teak
2013	30000	25000	15000
2014	35000	30000	5000
2015	35000	45000	10000

2016	40000	40000	25000
2017	45000	55000	35000
2018	55000	50000	40000

Q1. Find the respective ratio between the number of neem trees planted in the year 2015 and the number of banyan trees planted in the year 2014

Solution:
Number of neem trees planted in 2015 = 45000
Number of banyan trees planted in 2014 = 35000
Required ratio = 45000 : 35000 = 9 : 7

Q2. What was the approximate average number of neem trees planted in all the years together?

Solution:
Total number of neem trees planted in all the years = 25000 + 30000 + 45000 + 40000 + 55000 + 50000 = 245000
Required average = 245000/6 = 40833.33 ≈ 40830 (approx. depends on options given in question)

Q3. How much percent more teak trees planted by government in the year 2017 as compared to 2016?

Solution:
Total teak trees planted in year 2017 = 35000
Total teak tree planted in year 2016 = 25000
Percentage increase = (35000 - 25000)/25000 × 100 = 40%.

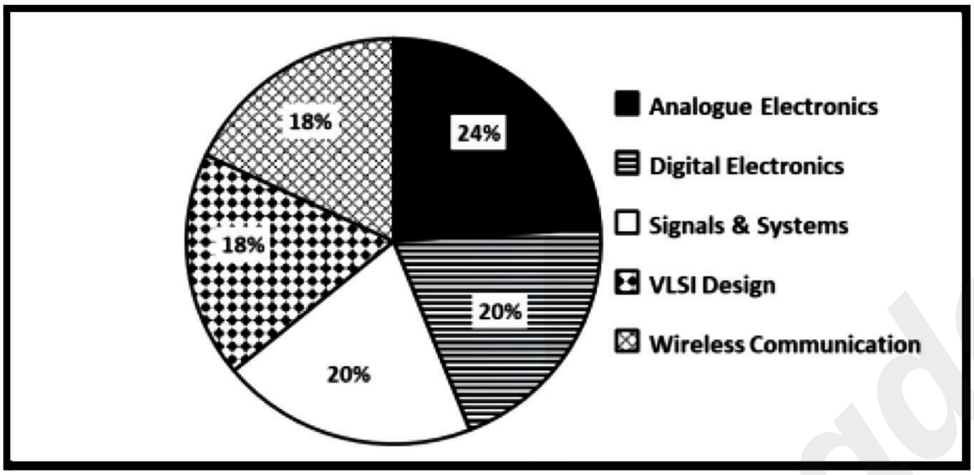
Pie Chart

It is a circular chart divided in various sectors. The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding values of information provided. In pie charts total quantity is distributed over a total angle of 360° or 100%.

Points to Remember

- Value of sector = (Angle of sector/360°) × Total Value of sector
- Value of sector = (Percentage of sector/100) × Total value

Directions: **The following Pie chart shows the percentage of students who like five different subjects of engineering from college x. Percentage of students who like 5 different subjects of engineering.**



Q1. What is the difference between the number of students who like Analogue Electronics and VLSI Design?

Solution:

Number of students liking Analogue Electronics = (1000/100) × 24 = 240.

Number of students liking VLSI Design = (1000/100) × 18 = 180.

Required difference = 240 – 180 = 60.

Q2. Find the total number of students who like Analogue Electronics, VLSI Design and Wireless Communication.

Solution:

Number of students liking Analogue Electronics = (1000/100) × 24 = 240.

Number of students liking VLSI Design = (1000/100) × 18 = 180.

Number of students liking Wireless Communication = (1000/100) × 18 = 180.

Total number of students = 240 + 180 + 180 = 600.

Q3. The number of students who like VLSI Design are how much percent less than the number of students who like Analogue Electronics?

Solution:

Number of students liking VLSI Design = (1000/100) × 18 = 180.

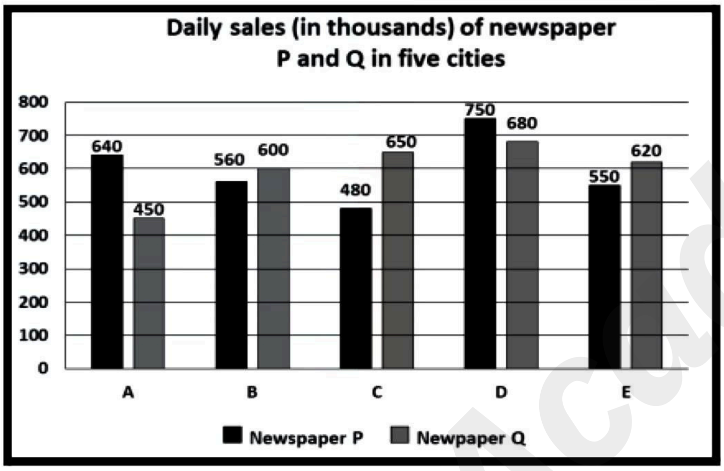
Number of students liking Analogue Electronics = (1000/100) × 24 = 240.

Difference of students = 240 – 180 = 60

Required percentage = (60/240) × 100 = 25%.

Bar Graph

Directions: **Study the given graph and answer the question that follows.**



Q1. What is the ratio of the total daily sales of newspaper P in cities A and C to the total daily sales of newspaper Q in cities B and D?

Solution:

Total sales of newspaper P in cities A and C = 640 + 480 = 1120

Total sales of newspaper Q in cities B and D = 600 + 680 = 1280

Required ratio = 1120:1280 = 7:8.

Q2. The total daily sales of newspaper P in cities, B, D and E is what percentage less than that of newspaper Q in cities A, C, D and E?

Solution:

Total sales of newspaper P in cities B, D and E = 560 + 750 + 550 = 1860

Total sales of newspaper Q in cities A, C, D and E = 450 + 650 + 680 + 620 = 2400

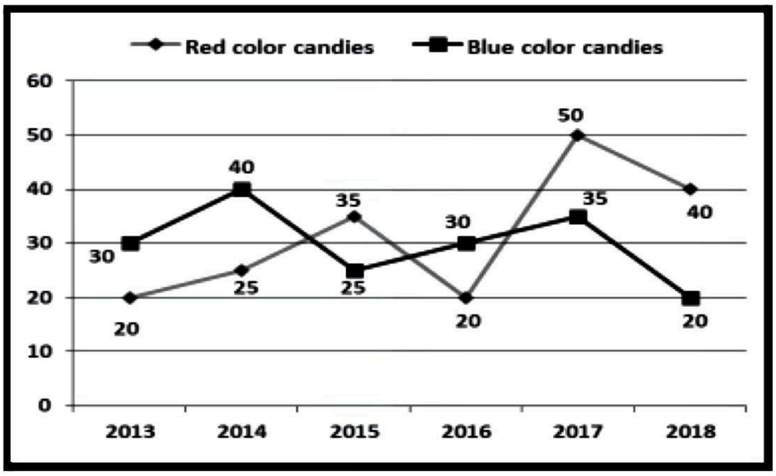
Required percentage = [(2400 – 1860)/2400] × 100 = 22.5%.

Line Graph

A line graph shows the quantitative information or a relationship between two changing quantities with a line or curve. We are required to understand the given information and thereafter answer the given questions.

Directions: **Read the following line graph carefully and answer the given questions below:**

Following line graph shows the number of red- and blue-coloured candies (in lakhs) produced in 6 different years.



Q1. Candies produced of red colour in 2018 are what percentage less/more than candies produced of red colour in 2017?

Solution:

Candies produced of red colour in 2018 = 40 lakhs

Candies produced of red colour in 2017 = 50 lakhs

Required percentage = (50 – 40)/50 × 100 = 20%.

Q2. Total candies produced in years 2014 and 2015 are how much more/less than total candies produced in years 2017 and 2018 together?

Solution:

Total candies produced in years 2014 and 2015 = 25 + 40 + 35 + 25 = 125 lakhs

Total candies produced in years 2017 and 2018 = 50 + 35 + 40 + 20 = 145 lakhs

Required difference = 145 – 125 = 20 lakhs.

Q3. For how many years candies produced of blue color are more or equal to average candies produced of blue color for all the years?

Solution:

Average number of blue color candies = (30 + 40 + 25 + 30 + 35 + 20)/6 = 180/6 = 30 lakhs.

Hence, required years = 2013, 2014, 2016, 2017.

Number of years = 4.

Q4. Find the difference between total number of red color candies and total number of blue color candies produced throughout the 6 years.

Solution:

Total number of red color candies = 20 + 25 + 35 + 20 + 50 + 40 = 190 lakhs
Total number of blue color candies = 30 + 40 + 25 + 30 + 35 + 20 = 180 lakhs
Required difference = 190 – 180 = 10 lakhs.

Q5. Find the ratio of the number of candies of red color produced in years 2013, 2014 and 2015 together to the number of candies of blue color produced in years 2016, 2017 and 2018 together.

Solution:
The number of candies of red color produced in years 2013, 2014 and 2015 together = 20 + 25 + 35 = 80 lakhs
The number of candies of blue color produced in years 2016, 2017 and 2018 together = 30 + 35 + 20 = 85 lakhs
Required ratio = 80/85 = 16: 17

Caselet DI

In Caselet DI, a long paragraph is provided and with that as the basis, some sets of questions are asked. We need to understand the given information and then answer the given questions.

Comprehension:

Three friends A, B and C went to a shopping mall to buy laptop (individually), they all are having same amount of money equal to the value of MRP of a Laptop (i.e., Rs. 60000) but each one of them got different discounts from the exclusive showroom. If A paid Rs. 51000, B has paid Rs. 3000 more than what A has paid while C has paid the amount equal to the average value of amount paid by A and B together. Find the answer of the following questions based on the given information:

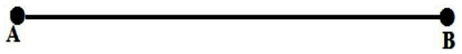
Q1. What is the ratio of the total amount paid by A, B and C together to the sum of MRP of all the three laptops?

Solution:
The MRP of each laptop = Rs. 60000;
The sum of MRP of 3 laptops = 60000 × 3 = Rs. 1,80,000
Now, amount paid by A = Rs. 51000
Amount paid by B = Rs. 51000 + 3000 = Rs. 54000
Amount paid by C = (51000 + 54000)/2 = Rs. 52500
The total amount paid by A, B and C together = 51000 + 54000 + 52500 = Rs. 1,57,500
The ratio of the total amount paid by A, B and C together to the sum of MRP of all the three laptops = 157500:180000 = 7:8

FUNDAMENTAL CONCEPTS OF GEOMETRY

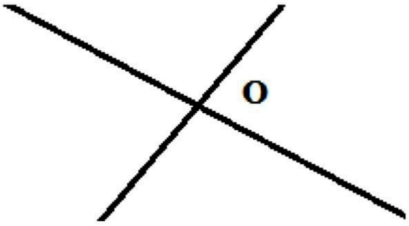
Point: It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has position i.e., it has no magnitude.

Line segment: The straight path joining two points A and B is called a line segment points and a definite length.

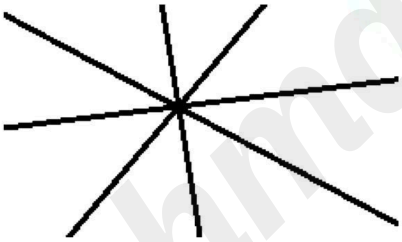


Ray: A line segment which can be extended in only one direction is called a ray.

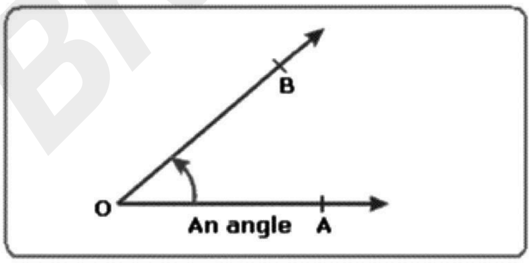
Intersecting lines: Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.



Concurrent lines: If two or more lines intersect at the same point, then they are known as concurrent lines.

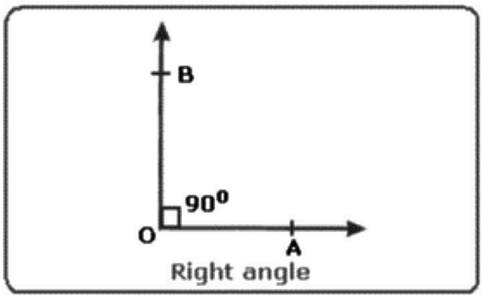


Angles: When two straight lines meet at a point they form an angle.

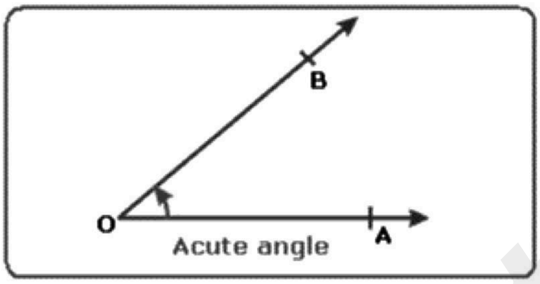


In the figure above, the angle is represented as ∠AOB. OA and OB are the arms of ∠AOB. Point O is the vertex of ∠AOB. The amount of turning from one arm (OA) to other (OB) is called the measure of the angle (∆ AOB).

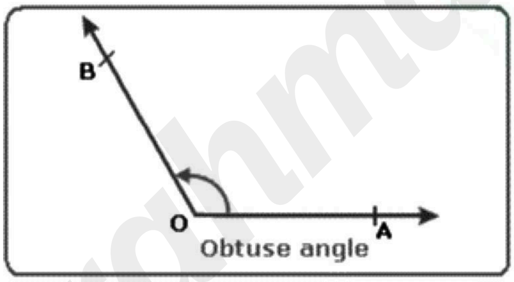
Right angle: An angle whose measure is 90 is called a right angle.



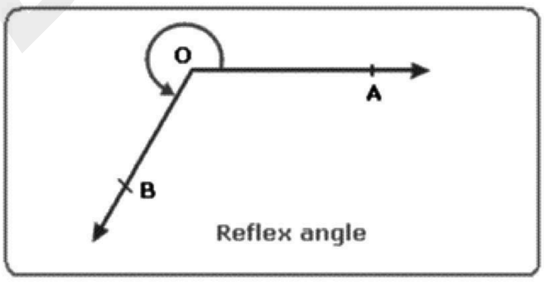
Acute angle: In angle whose measure is less than one right angle (i.e., less than 90), is called an acute angle.



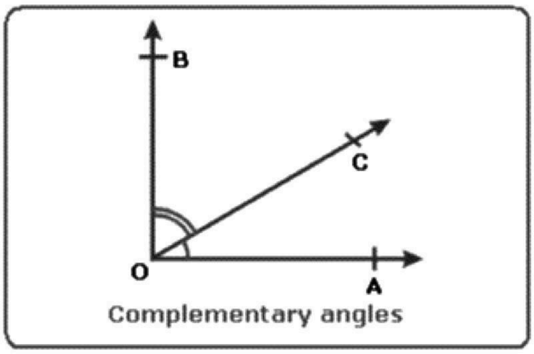
Obtuse angle: An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180 and more than 90) is called an obtuse angle.



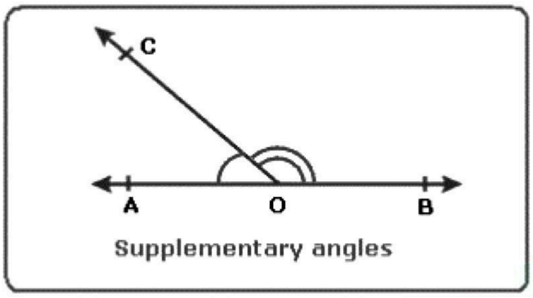
Reflex angle: An angle whose measure is more than 180 and less than 360 is called a reflex angle.



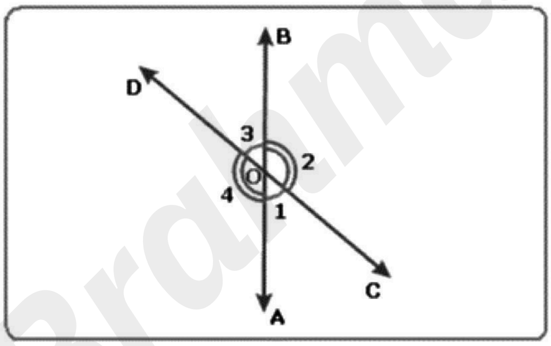
Complementary angles: If the sum of the two angles is one right angle (i.e., 90), they are called Complementary angles. Therefore, the complement of an angle θ is equal to 90° − θ.



Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180. Example: Angles measuring 130 and 50 are supplementary angles. Two supplementary angles are the supplement of each other. Therefore, the supplement of an angle θ is equal to 180° − θ.

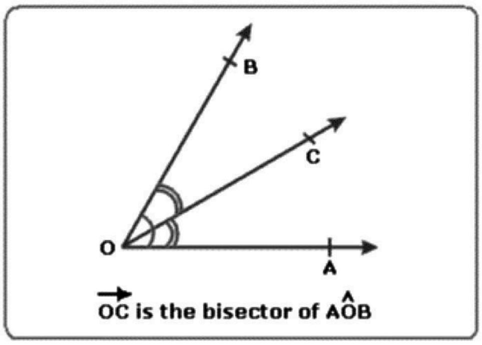


Vertically opposite angles: When two straight lines intersect each other at a point, the pairs of opposite angles so formed are called vertically opposite angles.

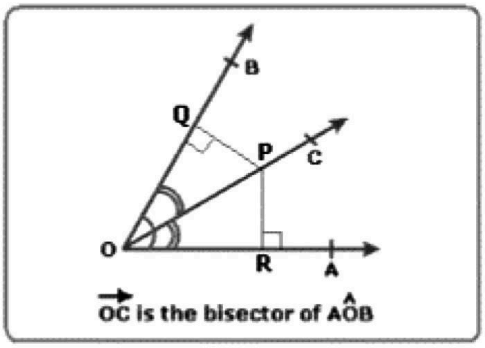


In the above figure, ∠1 and ∠3 and angles ∠2 and ∠4 are vertically opposite angles.
Note: Vertically opposite angles are always equal.

Bisector of an angle: If a ray or a straight line passing through the vertex of that angle, divides the angle into two angles of equal measurement, then that line is known as the Bisector of that angle.



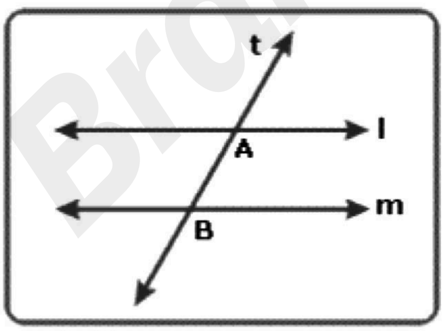
A point on an angle is equidistant from both the arms.



In the figure above, Q and R are the feet of perpendiculars drawn from P to OB and OA. It follows that PQ = PR.

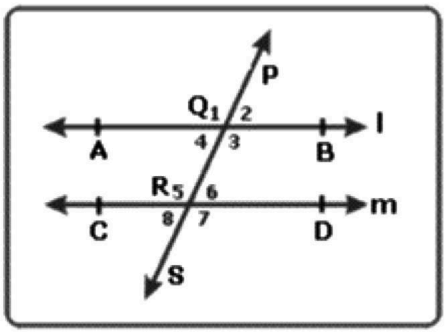
Parallel lines: Two lines are parallel if they are coplanar and they do not intersect each other even if they are extended on either side.

Transversal: A transversal is a line that intersects (or cuts) two or more coplanar lines at distinct points.



In the above figure, a transversal t is intersecting two parallel lines, l and m, at A and B, respectively.

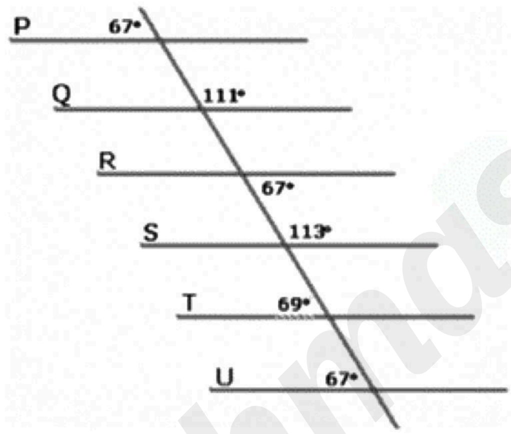
Angles formed by a transversal of two parallel lines:



In the above figure, l and m are two parallel lines intersected by a transversal PS. The following properties of the angles can be observed:

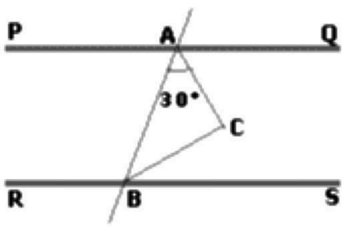
∠3 = ∠5 and ∠4 = ∠6 [Alternate angles]
∠1 = ∠5, ∠2 = ∠6, ∠4 = ∠8, ∠3 = ∠7 [Corresponding angles]
∠4 + ∠5 = ∠3 + ∠6 = 180° [Supplementary angles]

In the figure given below, which of the lines are parallel to each other?



Answer: As 67° + 113° = 180°, lines P and S, R and S, and S and U are parallel. Therefore, lines P, R, S and U are parallel to each other. Similarly, lines Q and T are parallel to each other.

Example: In the figure given below, PQ and RS are two parallel lines and AB is a transversal. AC and BC are angle bisectors of ∠BAQ and ∠ABS, respectively. If ∠BAC = 30°, find ∠ABC and ∠ACB.



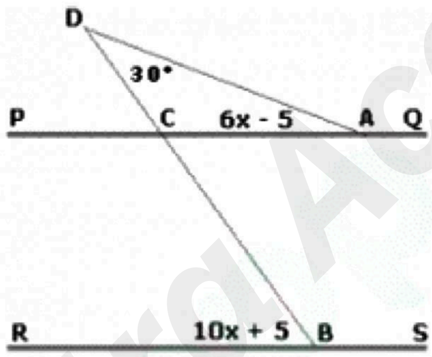
Answer:

∠BAQ + ∠ABS = 180° [Supplementary angles]

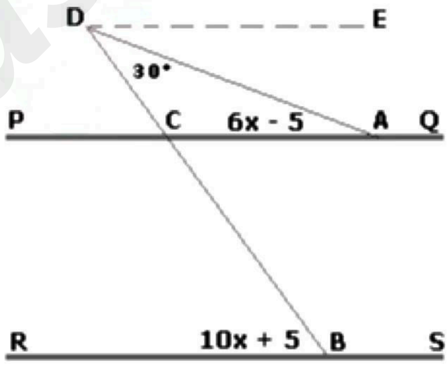
$\frac{\angle BAQ}{2} + \frac{\angle ABS}{2} = \frac{180}{2} = 90°$ => ∠BAC = ∠ABC = 90°

Therefore, ∠ABC = 60° and ∠ACB = 90°.

Example: For what values of x in the figure given below are the lines P-A-Q and R–B-S parallel, given that AD and BD intersect at D?



Answer: We draw a line DE, parallel to RS, as shown in the figure below:

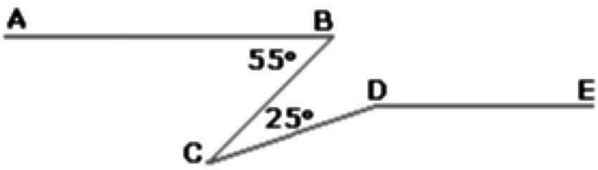


In the above figure, ∠CDE = ∠RBD = 10x + 5 ⇒ ∠CDA = 10x + 5 −30 = 10x − 25.

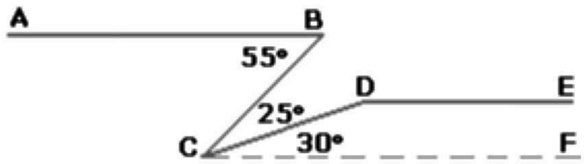
Let the line PQ and RS be parallel. Therefore, PQ // DE ⇒

∠EDA = ∠CAD = 10x − 25 = 6x − 5 ⇒ x =5

Example: In the figure given below, lines AB and DE are parallel. What is the value of ∠CDE?



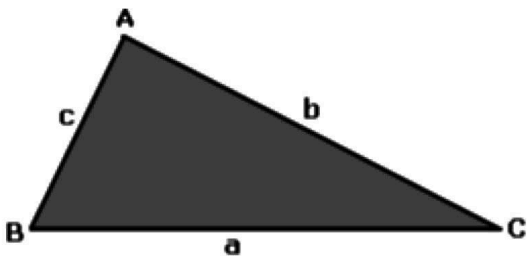
Answer: We draw a line CF // DE at C, as shown in the figure below.



∠BCF = ∠ABC = 55° ⇒ ∠DCF = 30°.
⇒ CDE = 180° − 30° = 150°.

TRIANGLE

Triangle is closed figures containing three angles and three sides.



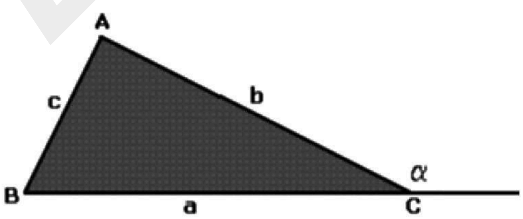
General Properties of Triangles:

1. **The sum of the two sides is greater than the third side: a + b > c, a + c > b, b + c > a**

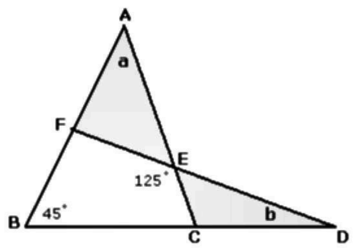
Example: The two sides of a triangle are 12 cm and 7 cm. If the third side is an integer, find the sum of all the values of the third side.

Answer: Let the third side be of x cm. Then, x + 7 > 12 or x > 5. Therefore, minimum value of x is 6. Also, x < 12 + 7 or x < 19. Therefore, the highest value of x is 18. The sum of all the integer values from 6 to 18 is equal to 156.

2. **The sum of the three angles of a triangle is equal to 180°: In the triangle below ∠A + ∠B + ∠C = 180°**

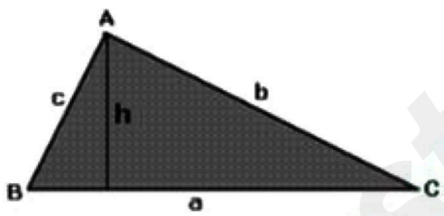


Also, the exterior angle α is equal to sum the two opposite interior angle A and B, i.e., α = ∠A + ∠B.
Example: Find the value of a + b in the figure given below:



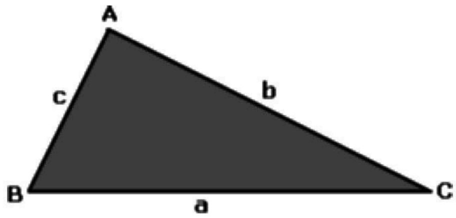
Answer: In the above figure, ∠CED = 180° − 125° = 55°.
∠ACD is the exterior angle of ΔABC. Therefore,
∠ACD = a + 45°. In ΔCED, a + 45° + 55° + b = 180°
⇒ a + b = 80°

3. Area of a Triangle:



Area of a triangle = ½ × base × height = ½ × a × h
Area of a triangle = ½ bc sin A = ½ ab sin C = ½ ac sin B
Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Where s = $\frac{a+b+c}{2}$
Area of triangle = $\frac{abc}{4R}$ where R = circumradius
Area of a triangle = r × s where r = in radius and s = $\frac{a+b+c}{2}$

4. More Rules:



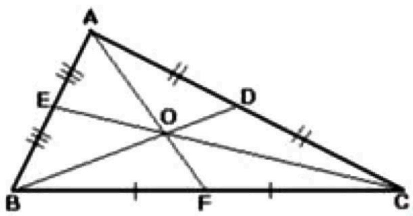
Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Rule:

$$\cos C = \frac{b^2 + c^2 + a^2}{2bc}$$

5. Medians of a triangle:



The medians of a triangle are lines joining a vertex to the midpoint of the opposite side. In the figure, AF, BD and CE are medians. The point where the three medians intersect is known as the centroid. O is the centroid in the figure.

The medians divide the triangle into two equal areas. In the figure, area ΔABF = area ΔAFC = area ΔBDC = area ΔBDA = area ΔCBE = area ΔCEA = $\frac{\text{Area } \Delta ABC}{2}$

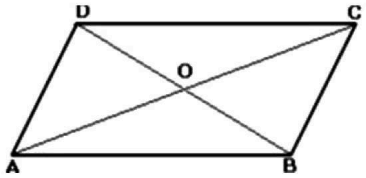
The centroid divides a median internally in the ratio 2: 1. In the figure, $\frac{AO}{OF} = \frac{BO}{OD} = \frac{CO}{OE}$

Apollonius Theorem:

$AB^2 + AC^2 = 2(AF^2 + BF^2)$
$BC^2 + BA^2 = 2(BD^2 + DC^2)$
$BC^2 + AC^2 = 2(EC^2 + AE^2)$

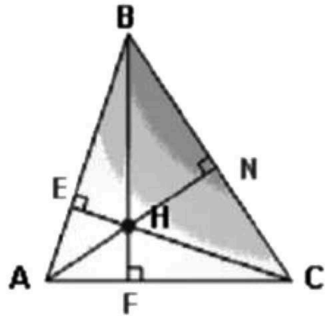
Example: ABCD is a parallelogram with AB = 21 cm, BC = 13 cm and BD= 14 cm. Find the length of AC.

Answer: The figure is shown below. Let AC and BD intersect at O. O bisects AC and BD. Therefore, OD is the median in triangle ADC



⇒ $AD^2 + CD^2 = 2(AO^2 + DO^2)$ ⇒ AO = 16. Therefore, AC = 32.

6. Altitudes of a Triangle:

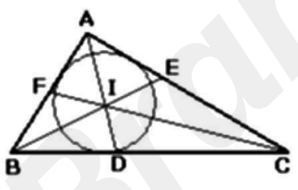


The altitudes are the perpendiculars dropped from a vertex tothe opposite side. In the figure, AN, BF, and CE are the altitudes, and their point of intersection, H, is known as the orthocenter.

Triangle ACE is a right-angled triangle. Therefore, ∠ECA = 90° − ∠A. Similarly in triangle CAN, ∠CAN = 90° − ∠C. In triangle AHC, ∠CHA = 180° − (∠HAC + ∠HCA) = 180° − (90° − ∠A + 90° − ∠C) = ∠A + ∠C = 180° − ∠B.

Therefore, ∠AHC and ∠B are supplementary angles.

7. Internal Angle Bisectors of a Triangle:



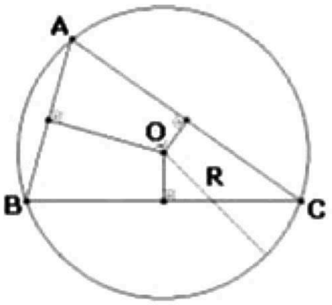
In the figure above, AD, BE and CF are the internal angle bisectors of triangle ABC. The point of intersection of these angle bisectors, I, is known as the incentre of the triangle ABC, i.e., centre of the circle touching all the sides of a triangle.

∠BIC = 180° − (∠IBC + ∠ICB)

$= \left(\frac{B}{2} + \frac{C}{2}\right) = 180 - \left(\frac{B+C}{2}\right) = 180 - \left(\frac{180-A}{2}\right) = 90 + \frac{A}{2}$

$\frac{AB}{AC} = \frac{BD}{CD}$ (internal bisector theorem)

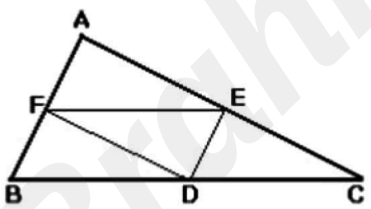
8. Perpendicular Side Bisectors of a Triangle:



In the figure above, the perpendicular bisectors of the sides AB, BC and CA of triangle ABC meet at O, the circumcentre (centre of the circle passing through the three vertices) of triangle ABC. In Above figure, O is the centre of the circle and BC is a chord. Therefore, the angle subtended at the centre by BC will be twice the angle subtended anywhere else in the same segment.

Therefore, ∠BOC = 2∠BAC.

9. Line Joining the Midpoints:



In the figure above, D, E and F are midpoints of the sides of triangle ABC. It can be proved that:

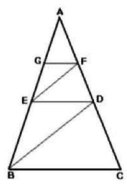
FE // BC, DE // AB and DF // AC.

$FE = \frac{BC}{2}$, $DE = \frac{AB}{2}$, $FD = \frac{AC}{2}$

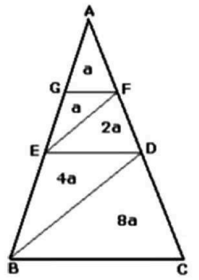
Area ΔDEF = Area Δ AFE = Area Δ BDF = Area Δ DEC

$= \frac{\text{Area } \Delta ABC}{4}$

Example: In the figure given below: AG = GE and GF || ED, EF || BD and ED || BC. Find the ratio of the area of triangle EFG to trapezium BCDE.

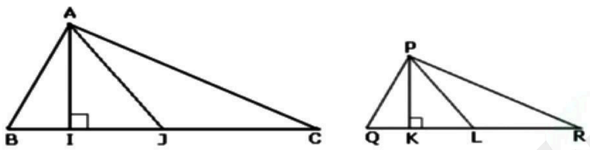


Answer: We know that line parallel to the base and passing through one midpoint passes through another midpoint also. Using this principle, we can see that G, F, E and D are midpoints of AE, AD, AB, and AC respectively. Therefore, GF, EF, ED, and BD are medians in triangles AFE, AED, ADB and ABC.



We know that medians divide the triangle into two equal areas.
Let the area of triangle AGF = a.
Therefore, the areas of the rest of the figures are as shown above.
The required ratio = a/12a = 1/12.

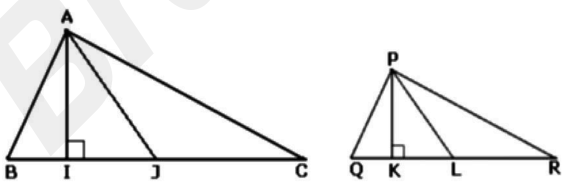
Similarity of Triangles



Two triangles are similar if their corresponding angles are equal or corresponding sides are in proportion.
In the figure given above, triangle ABC is similar to triangle PQR.
Then ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R and
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AI}{PK}$ (altitudes) $= \frac{AJ}{PL}$ (medians)

Therefore, if you need to prove two triangles similar, prove their corresponding angles to be equal or their corresponding sides to be in proportion.

Ratio of Areas



If two triangles are similar, the ratio of their areas is the ratio of the squares of the length of their corresponding sides. Therefore,

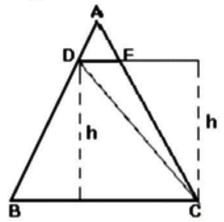
$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

Example: In triangle AC, shown above, DE || BC and DE/BC = ¼. If area of triangle ADE is 10, find the area of the trapezium BCED and the area of the triangle CED.

Answer: ∆ADE and ∆ABC are similar. Therefore,

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle ADE}} = \frac{BC^2}{DE^2}$$

Area of triangle ABC = 160 => Area of trapezium BCDE = Area ∆ ABC – Area ∆ ADE = 160 – 10 = 150



To find the area of triangle CDE, we draw altitudes of triangle BDC and CDE, as shown above. Let the length of the altitudes be h.

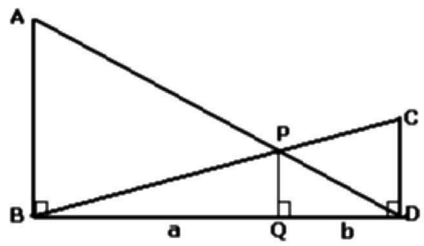
Area of triangle BCD = ½ x BC x h and area of triangle CDE = ½ x DE x h

- ⇨ $\frac{\text{Area of triangle BCD}}{\text{Area of triangle CDE}} = \frac{BC}{DE} = 4$

Therefore, we divide the area of the trapezium BCED in the ratio 1: 4 to find the area of triangle CDE.

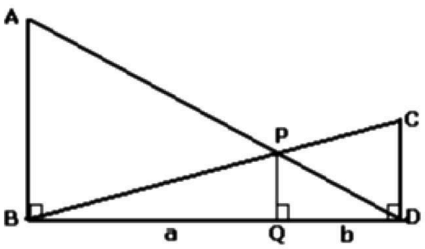
The required area = 1/5 x 150 = 30

Example: In the diagram given below, ∠ABD = ∠CDB = ∠PQD = 90°. If AB: CD = 3: 1, the ratio of CD: PQ is-



(A) 1 : 0.6 (B) 1 : 0.75
(C) 1: 0.72 (D) 1: 0.77

Answer: Let BQ = a and DQ = b, as shown in the figure below.



Triangle ABD and triangle PQD are similar. Therefore,

$\frac{PQ}{AB} = \frac{b}{a+b}$

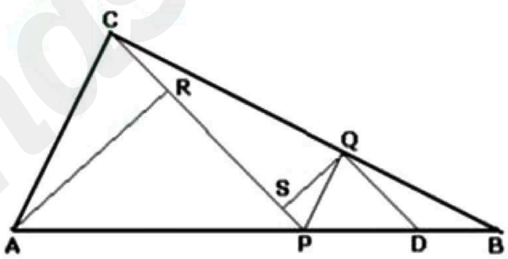
Also, triangle CBD and triangle PBQ are similar, therefore

$\frac{PQ}{CD} = \frac{a}{a+b}$

Dividing the second equality by the first, we get, $\frac{AB}{CD} = \frac{a}{b} = 3$,

Therefore, $\frac{CD}{PQ} = \frac{a+b}{a} = \frac{4}{3} = 1 : 0.75$

Example:



In the figure (not drawn to scale) given below, P is a point on AB such that AP: PB = 4: 3. PQ is parallel to AC and QD is parallel to CP. In △ARC, ∠ARC = 90°, and in △PQS, ∠PSQ = 90°. The length of QS is 6 cm. What is ratio AP: PD?

(A) 10 : 3 (B) 2 : 1
(C) 7 : 3 (D) 8 : 3

Answer: PQ is parallel to AC

$\frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$

Let AP = 4x and PB = 3x.

QD is parallel CP ⇒ $\frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$ ⇒ PD = $\frac{4PB}{7} = \frac{12x}{7}$

⇒ AP: PD = 4x: $\frac{12x}{7}$ = 7 : 3

Example: In the figure (not drawn to scale given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In ∆ ARC, ∠ARC = 90°, and in ∆ PQS, ∠PSQ = 90°. The length of QS is 6 cm. What is ratio AP : PD?

(A) 10 : 3 (B) 2 : 1
(C) 7 : 3 (D) 8 : 3

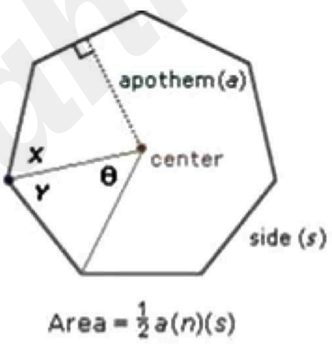
Answer: PQ is parallel to AC ⇒ $\frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$

Let AP = 4 × and PB = 3x.
QD is parallel CP ⇒ $\frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

PD = $\frac{4PB}{7} = \frac{12x}{7}$ ⇒ 4x : $\frac{12x}{7}$ = 7 : 3

Regular Polygon

A regular polygon is a polygon with all its sides equal and all its interior angles equal. All vertices of a regular lie on a circle whose centre is the center of the polygon.



Each side of a regular polygon subtends an angle $\Theta = \frac{360}{n}$ at the centre, as shown in the figure.

Also, X = Y = $\frac{180-\frac{360}{n}}{2} = \frac{\{180\ (n-2)\}}{2n}$

Therefore, interior angle of a regular polygon = x + y = $180(n - 2)/2n$

Sum of all angles of a regular polygon = $n\ x \frac{\{180\ (n-2)\}}{n} = 180\ (n - 2)$

Example: What is the interior angle of a regular octagon?

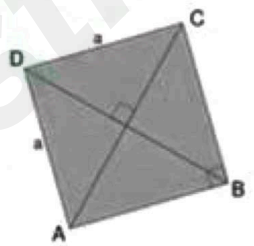
Answer: The interior angle of a regular octagon = n x 180 (n-2)/n = 180 (n-2)

Note: The formula for sum of all the angle of a regular polygon, i.e., 180(n-2), is true for all n-sided convex simple polygons.

Let's look at some polygons, especially quadrilaterals:

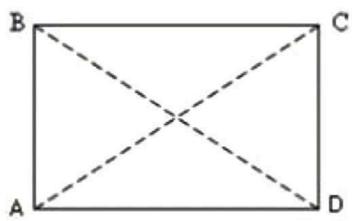
Quadrilateral: A quadrilateral is any closed shape that has four side. the sum of the measures of the angle is 3600. Some of the known quadrilaterals are square, rectangle, trapezium, parallelogram and rhombus.

Square: A square is regular quadrilateral that has four right angles and parallel sides. The sides of a square meet at right angles. The diagonal also bisect each other perpendicularly.



If the side of the square is a, then its perimeter = 4a, area = a^2 and the length of the diagonal = $\sqrt{2}$a

Rectangle: a rectangle is a parallelogram with all its angles equal to right angles.

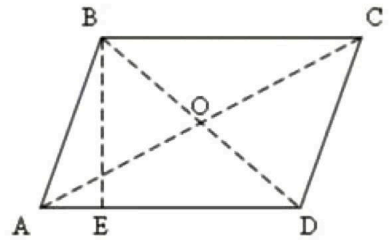


Properties of a rectangle:

- Sides of rectangle are its heights simultaneously.
- Diagonals of a rectangle are equal: AC = BD.

- A square of a diagonal length is equal to a sum of squares of its side's lengths, i.e. $AC^2 = AD^2 + DC^2$.
- Area of a rectangle = length × breadth

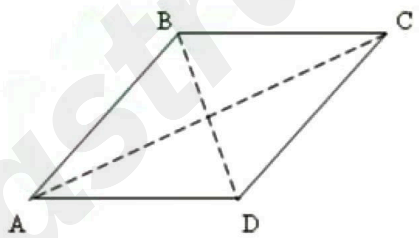
Parallelogram: A parallelogram is a quadrangle in which opposite sides are equal and parallel.



- Any two opposite sides of a parallelogram are called bases, a distance between them is called a height.
- Area of a parallelogram = base × height
- Perimeter = 2(sum of two consecutive sides)

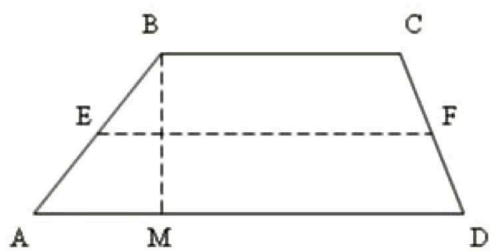
Properties of a parallelogram:
- Opposite side of a parallelogram are equal (AB = CD, AD = BC).
- Opposite angles of a parallelogram are equal (∠A = ∠C, ∠B = ∠D).
- Diagonals of a parallelogram are divided in their intersection point into two (AO = OC, BO = OD).
- A sum of squares of diagonals is equal to a sum of squares of four sides:
  ACs + BDs = ABs + BCs + CDs + ADs.

Rhombus: If all sides of parallelogram are equal, then this parallelogram is called a rhombus.



- Diagonals of a rhombus are mutually perpendicular (AC ⊥ BD) and divide its angle into two (∠DCA = ∠BCA, ∠ABD = ∠CBD etc.).
- Area of a rhombus = ½ × product of diagonals
  = ½ × AC × BD

Trapezoid: Trapezoid is a quadrangle two opposite sides of which are parallel.



Here AD || BC. Parallel sides are called bases of a trapezoid, the two others (AB and CD) are called lateral sides. A distance between bases (BM) is a height. The segment EF, joining midpoints E and F of the lateral sides, is called a midline of a trapezoid. A midline of a trapezoid is equal to a half-sum of bases:

$$EF = \frac{AD + BC}{2}$$

and parallel to them: EF || AD and EF || BC.

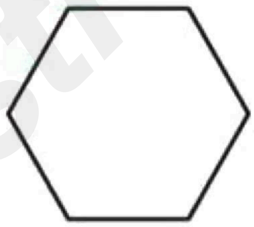
A trapezoid with equal lateral sides (AB = CD) is called an isosceles trapezoid. In an isosceles trapezoid angle by each base, are equal (∠A = ∠D, ∠B = ∠C).

Area of a trapezoid = $\frac{\text{Sum of parallel sides}}{2}$ x height = $\frac{AD + BC}{2}$ x BM

In a trapezium ABCD with bases AB and CD, the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the non-parallel sides and twice the product of the lengths of the parallel sides: AC2 + BD2 = AD2 + BC2 + 2.AB.CD

Here is one more polygon, a regular hexagon:

Regular Hexagon: A regular hexagon is a closed figure with six equal sides.

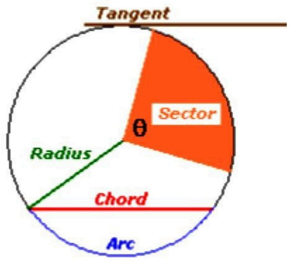


If we join each vertex to the centre of the hexagon, we get 6 equilateral triangles. Therefore, if the side of the hexagon is a, each equilateral triangle has a side a. Hence, area of the regular hexagon:

$$6 \times \frac{\sqrt{3}}{4} a^2 = 3\frac{\sqrt{3}}{2} a^2$$

Circle

A circle is a set of all points in a plane that lie at a constant distance from a fixed point. The fixed point is called the center of the circle and the constant distance is known as the radius of the circle.



Arc: An arc is a curved line that is part of the circumference of a circle. A minor arc is an arc less than the semicircle and a major arc is an arc greater than the semicircle.

Chord: A chord is a line segment within a circle that touches points on the circle.

Diameter: The longest distance from one end of a circle to the other is known as the diameter. It is equal to twice the radius.

Circumference: The perimeter of the circle is called the circumference. The value of the circumference = 2πr, where r is the radius of the circle.

Area of a circle sector = $\pi \times (\text{radius})^2 = \pi r^2$

Sector: A sector is like a slice of pie (a circular wedge).
Area of circle sector (with central angle ϴ);

$$\text{Area} = \frac{\Theta}{360} \times \pi \times r^2$$

Length of a circular Arc: (with central angle ϴ) = $\frac{\Theta}{360} \times 2 \pi \times r$

Tangent of circle: A line perpendicular to the radius that touches ONLY one point on the circle.

Example: If 45° arc of circle A has the same length as 60° arc of circle B, find the ratio of the areas of circle A and circle B.

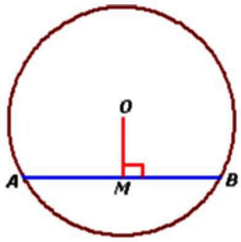
Answer: Let the radius of circle A be r_1 and that of circle B be r_2.

- $\frac{45}{360} \times 2 \pi \times r_1 = \frac{60}{360} \times 2 \pi \times r^2$

- Ratio of areas = $\frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{9}$

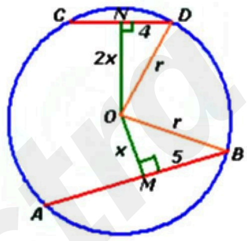
Rule:

The perpendicular from the center of a circle to a chord of the circle bisects the chord. In the figure below, O is the centre of the circle and OM ⊥ AB, Then, AM = MB.



Conversely, the line joining the center of the circle and the midpoint of a chord is perpendicular to the chord.

Example: In a circle, a chord of length 8 cm is twice as far from the center as a chord of length 10 cm. Find the circumference of the circle.



Answer: Let AB and CD be two chords of the circle such that AB = 10 and CD = 8.

Let O be the center of the circle and M and N be the midpoints of AB and CD.

Therefore OM ⊥ AB, ON ⊥ CD, and if ON = 2x then OM = x.

$BM^2 + OM^2 = OB^2$ and $DN^2 + ON^2 = OD^2$.

OB = OD = r → $(2x)^2 + 4^2\ r^2$ and $x^2 + 5^2 = r^2$.

Equating both the equations we get, $4x^2 + 16 = x^2 + 25$

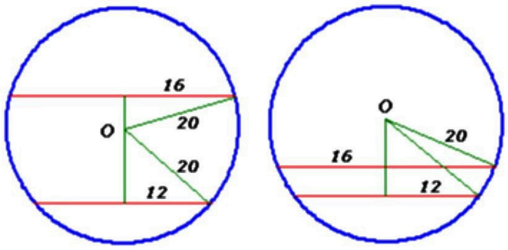
Or $x\sqrt{3} \rightarrow 2\sqrt{7}$

Therefore, circumference is = 2 πr = 4 $\pi\sqrt{7}$

Example: What is the distance in cm between two parallel chords of length 32 cm and 24 cm in a circle of radius 20 cm?

(A) 1 or 7 (B) 2 or 14
(C) 3 or 21 (4) 4 or 28

Answer: The figures are shown below:



The parallel chords can be on the opposite side or the same side of the centre O. The perpendicular (s) dropped on the chords from the centre bisect (s) the chord into segments of 16 cm and 12 cm, as shown in the figure. From the Pythagoras theorem, the distances of the chords from the centre are –

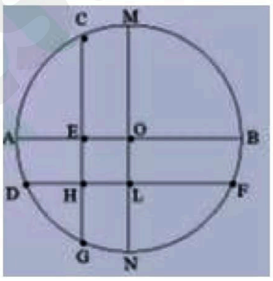
$\sqrt{}$ = 12 $\sqrt{}$ = 16, respectively.

$20^2 - 12^2$

$20^2 - 16^2$

Therefore, the distances between the chords can be 16 + 12 = 28 cm or 16 – 12 = 4 cm.

Example: In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2, and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is



Answer: In the above figure, AB = MN = 3 cm and AE: EB = NL: LM = 1: 2

⇒ AE = NL = 1 cm. Now AO = NO = 1.5 cm

⇒ OE = HL = OL

⇒ 0.5 cm. Join O and D

⇒ $OD^2 = OL^2 + DL^2$

⇒ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ = =

2

$1.5^2 - 0.5^2$

$OD^2 - OL^2$

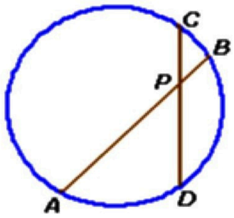
⇒ $\sqrt{}$DH = DL – HL = $- \frac{1}{2}$ = $\sqrt{}$ 2 2 - 1

2

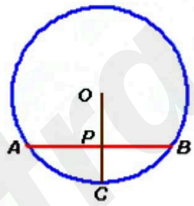
Some Important Rules

Rule #1
Equal chords are equidistant from the center. Conversely, if two chords are equidistant from the center of a circle, they are equal.

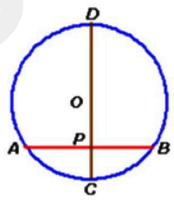
Rule #2
In the following figure, two chords of a circle, AB and CD, intersect at point P. Then, AP × PB = CP × PD.



Example: In the following figure, length of chord AB = 12. O-P-C is a perpendicular drawn to AB from center O and intersecting AB and the circle at P and C respectively. If PC = 2, find the length of OB.



Answer: Let us extend OC till it intersects the circle at some point D.



D is the diameter of the circle. Since OP is perpendicular to AB, P is the midpoint of AB.

Hence, AP = PB = 6.

Now DP × PC = AP × PB

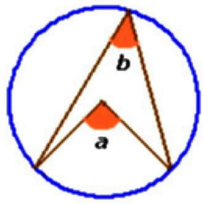
DP = 18. Therefore, CD = 20, OC = 10

OB = OC = radius of the circle = 10.

Rule #3
In a circle, equal chords subtend equal angles at the center.

Rule #4

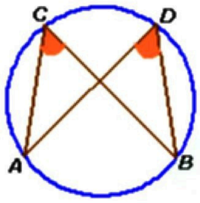
The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circumference.



In the figure shown above, a = 2b.

Rule #5

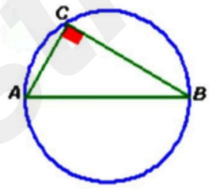
Angles inscribed in the same arc are equal.



In the figure angle ACB = angle ADB.

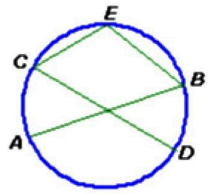
Rule #6

An angle inscribed in a semi-circle is a right angle.



Let angle ACB be inscribed in the semi-circle ACB; that is, let AB be a diameter and let the vertex C lie on the circumference; then angle ACB is a right angle.

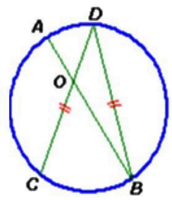
Example: In the figure AB and CD are two diameters of the circle intersecting at an angle of 48°. E is any point on arc CB. Find angle CEB.



Answer: Join E and D. Since arc BD subtends an angle of 48° at the center, it will subtend half as many degrees on the remaining part of circumference as it subtends at the center. Hence, angle DEB = 24°.

Since angle CED is made in a semicircle, it is equal to 90°. Hence, angle CEB = angle CED + angle DEB = 90° + 24° = 114°.

Example:



In the above figure, AB is a diameter of the circle and C and D are such points that CD = BD. AB and CD intersect at O. If angle AOD = 45°, find angle ADC.

Answer: Draw AC and CB.

CD = BD ⇒ ∠DCB = ∠DBC = θ (say).

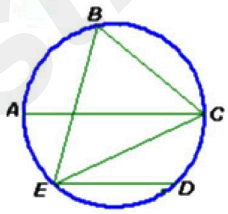
∠ACB = 90° ⇒ ∠ACD = 90°− θ.

∠ABD = ∠ACD = 90°− θ ⇒ ∠ABC = θ − (90°− θ) = 2θ − 90.

In ∆OBC, 45° + 2θ − 90 + θ = 180° ⇒ 3θ = 225° ⇒ θ = 75°.

∠ADC = ∠ABC = 2θ − 90 = 60°.

Example: In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If angle CBE = 65°, then what is the value of angle DEC?



(A) 35° (B) 55°
(C) 45° (D) 25°

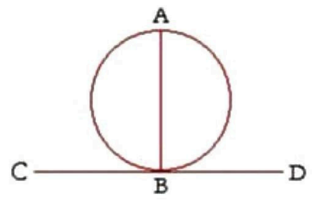
Answer: ∠ABC = 90° ⇒ ∠ABE = 90 − ∠EBC = 25°.

∠ABE = ∠ACE = 25°

∠ACE = ∠CED = 25° (alternate angles)

Rule #7

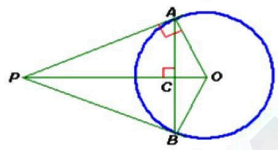
The straight line drawn at right angles to a diameter of a circle from its extremity is tangent to the circle. Conversely, if a straight line is tangent to a circle, then the radius drawn to the point of contact will be perpendicular to the tangent.



Let AB be a diameter of a circle, and let the straight line CD be drawn at right angles to AB from its extremity B; then the straight line CD is tangent to the circle.

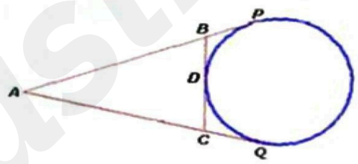
Rule #8

If two tangents are drawn to a circle from an exterior point, the length of two tangent segments are equal. Also, the line joining the exterior point to the centre of the circle bisects the angle between the tangents.



In the above figure, two tangents are drawn to a circle from point P and touching the circle at A and B. Then, PA = PB. Also, ∠APO = ∠BPO. Also, the chord AB is perpendicular to OP.

Example: In the following figure, lines AP, AQ and BC are tangent to the circle. The length of AP = 11. Find the perimeter of triangle ABC.

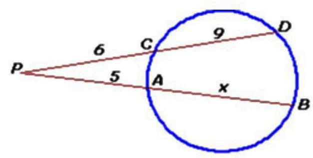


Answer: Let AB = x and BP = y. Then, BD = BP because they are tangents drawn from a same point B. Similarly, CD = CQ and AP = AQ.
Now perimeter of triangle ABC = AB + BC + CA = AB + BD + DC + AC
= AB + BP + CQ + AC = AP + AQ = 2AP = 22.

Rule #9

From an external point P, a secant P-A-B, intersecting the circle at A and B, and a tangent PC are drawn. Then, PA × PB = PC2.

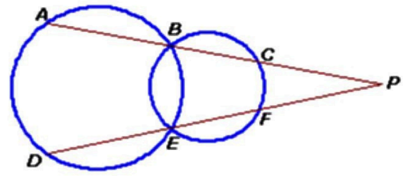
Example: In the following figure, if PC = 6, CD = 9, PA = 5 and AB = x, find the value of x.



Answer: Let a tangent PQ be drawn from P on the circle.
Hence, PC × PD = PQ = PA2 × PB = 6 × 15 = 5× (5 + x)

- ⇨ x = 13

Example: In the following figure, PC = 9, PB = 12, PA = 18, and PF = 8. Then, find the length of DE.



Answer: In the smaller circle
PC × PB = PF × PE

$PE = 12 \times \frac{9}{8} = \frac{27}{2}$

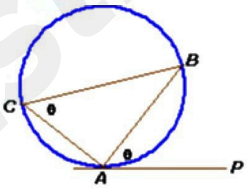
In the larger circle, PB × PA = PE × PD

$PD = 12 \times 18 \times \frac{2}{27} = 16$

Therefore, DE = PD – PE = 16 – 13.5 = 2.5

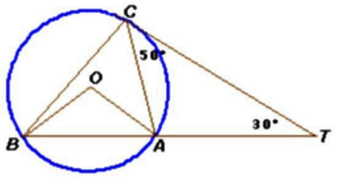
Rule #10

The angle that a tangent to a circle makes with a chord drawn from the point of contact is equal to the angle subtended by that chord in the alternate segment of the circle.



In the figure above, PA is the tangent at point A of the circle and AB is the chord at point A. Hence, angle BAP = angle ACB.

Example: In the figure given below, A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is –



(A) 100°   (B) 150°
(C) 80°   (D) Not possible to determine

Answer: Tangent TC makes an angle of 50° with chord AC.

Therefore, ∠TBC = 50°.

In triangle TBC,

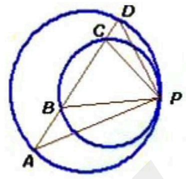
∠BCT = 180° − (30° + 50°) = 100°.

Therefore,

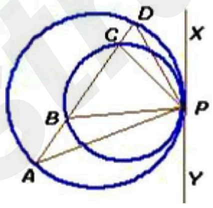
∠BCA = ∠BCT − ∠ACT = 100° − 50° = 50°.

∠BOA =2∠BCA = 100°.

Example: Two circles touch internally at P. The common chord AD of the larger circle intersects the smaller circle in B and C, as shown in the figure. Show that, ∠APB = ∠CPD.



Answer: Draw the common tangent XPY at point P.



Now, for chord DP, ∠DPX = ∠DAP, and for chord PC, ∠CPX = ∠CBP.

⇒ ∠CPD = ∠CPX − ∠DPX = ∠CBP − ∠DAP.

In triangle APB, ∠CBP is the exterior angle

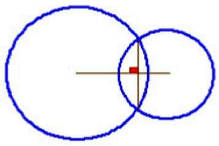
⇒ ∠CBP = ∠CAP + ∠APB

⇒ ∠CBP − ∠CAP = ∠APB

⇒ ∠CPD = ∠CPX − ∠DPX = ∠CBP − ∠DAP = ∠APB

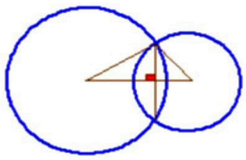
Rule #11

When two circles intersect each other, the line joining the centers bisects the common chord and is perpendicular to the common chord.



In the figure given above, the line joining the centers divides the common chord in two equal parts and is also perpendicular to it.

Example: Two circles, with diameters 68 cm and 40 cm, intersect each other and the length of their common chord is 32 cm. Find the distance between their centers.



Answer: In the figure given above, the radii of the circles are 34 cm and 20 cm, respectively. The line joining the centers bisects the common chord. Hence, we get two right triangles: one with hypotenuse equal to 34 cm and height equal to 16 cm, and the other with hypotenuse equal to 20 cm and height equal to 16 cm. Using Pythagoras theorem, we get the bases of the two right triangles equal to 30 cm and 12 cm. Hence, the distance between the centers = 30 + 12 = 42 cm.

PERCENTAGE

The Percentage is a fraction whose denominator is always 100. The sign of percentage is **%**.

Example:

10% can be converted to a fraction as 10/100 = 0.1

If, we want to calculate y% of x, then

Percentage Formula: y% of x = x × $\frac{y}{100}$

Question.

If 40% of P = 100, then find the value of P.

Ans. P × 40/100 = 100

⇒ P = 100 × 100/40

⇒ P = 250.

Fractions and Percentages

To express x% as a fraction

X% = $\frac{x}{100}$

Thus, 30% = 30/100 = 3/10

20% = 20/100 = 1/5

To express $\frac{a}{b}$ as a percent

We have $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$

Thus $\frac{1}{5}$ = ($\frac{1}{5}$ ×100) % = 20%

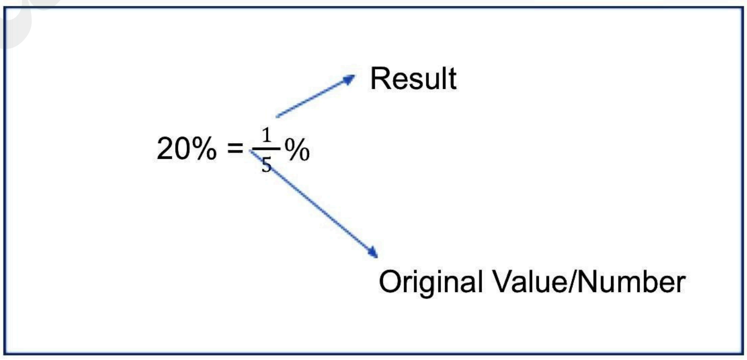


Fig: Representation and Interpretation of % in the form of Fraction.

Important Fraction to Percentage Conversions

$\frac{1}{2}$	**50%**
$\frac{1}{3}$	**33.33%**
$\frac{1}{4}$	**25%**
$\frac{1}{5}$	**20%**
$\frac{1}{6}$	**16.66%**
$\frac{1}{7}$	**14.28%**
$\frac{1}{8}$	**12.5%**
$\frac{1}{9}$	**11.11%**
$\frac{1}{10}$	**10%**
$\frac{1}{11}$	**9.09%**
$\frac{1}{12}$	**8.33%**
$\frac{1}{13}$	**7.69%**
$\frac{1}{14}$	**7.14%**
$\frac{1}{15}$	**6.66%**
$\frac{1}{16}$	**6.25%**
$\frac{1}{20}$	**5%**
$\frac{1}{25}$	**4%**

Basic Concepts of Percentages

Expressing One Quantity as a Percent with respect to the other:

To express a quantity as a percent with respect to other quantity, the following formula is used:

$$\left(\frac{\text{The quantity to be expressed in percent}}{\text{2nd quantity (in respect of which the percent has to be obtained)}} \times 100\right)\%$$

Calculation of Percentage

1. To express x% as a fraction:

We know

x% = x/100

Thus 10% = 10/100 (means 10 parts out of 100 parts)

= 1/10 (means 1 part out of 10 parts)

2. To express x/y as a percentage:

We know that x /y = (x /y× 100)

Thus 1/4= (1/4 ×100) % = 25%

and 0.8 = (8/10 ×100) % = 80%

3. To increase a number by a given percentage(x%): Multiply the number by the following factor

=$\left(\frac{100+x}{100}\right)$

4. To decrease a number by a given percentage(x%): Multiply the number by the following factor

=$\left(\frac{100-x}{100}\right)$

5. To find the % increase of a number:

Percent Increment= $\left(\frac{Final\ Value - Initial\ Value}{Initial\ value} \times 100\right)$

6. To find the % decrease of a number:

Percent Decrement = $\left(\frac{Initial\ \ Value - Final\ Value}{Initial\ value} \times 100\right)$

Some Observations

(1) If 20% candidate failed in an exam then observations are

· 80% represent passed in exam

· 100% represent total appeared in exam

· (80%-20%) = 60% represent difference between passed and failed candidate in exam

(2) If a number is increased by 25% then observations are

· 100% represent the old number

· 125% represent the new number.

(3) Remember that Base in the given sentence (Question) is always 100%

E.g., Income of Ram is increased by 20%

In this sentence

100% - represent the income of Ram

20% - represent increment

120% - represent new income of Ram.

(4) If of A is equal to y% of B then –

Z% of A = $(\frac{yz}{x})$% of B

(5) If A is more than B, then B is ($\frac{X}{100+X}\times100)$% less than A.

If A is X% less than B, then B is (($\frac{X}{100-X}\times100)$% more than A.

(6) If the passing marks in an examination is P%. If a candidate scores S marks and fails by F marks then

MM= $\frac{100\times[R+5]}{P}$

(7) If a candidate scores marks and fails by a mark while another candidate scores y% marks and gets b marks more than minimum passing marks, then –

Maximum Marks =$\frac{Sum\ of\ Scores}{Difference\ in\ \%\ marks}\times100$

(8) If due to decrement in the price of an item, a person can buy Kg more in y rupees, then actual price of that item –

= $\frac{(Rate)\times y}{(100-Rate)\times X}\,per\ Kg$

(9) If in an election, a candidate got of total votes cast and still lose by y votes, the total number of votes cast –

= $\frac{100\ \times X}{100-2X}$

(10) If the population of a town is P and it increases or decreases at the rate of R% per annum then –

I. Population after 'n' years:

= $P\ \times(1\pm\frac{R}{100})^{2}$

II. Population 'n' years ago:

=$\frac{p}{\left[1\pm\frac{R}{100}\right]^{n}}$

PROBLEM TYPE-1:

QUESTIONS BASED ON QUANTITY PURCHASED

Example: A reduction of 21% in the price of an item enables a person to buy 3 kg more for 100. The reduced price of item per kg is?

(a) Rs. 5.50

(b) Rs. 7.50

(c) Rs. 10.50

(d) Rs. 7.00

Solution:(d)

Reduced price will be:

Rp/100y per kg

In our case R= Rs. 100, x=21%, y=3kg

{(100 x 21)/ (100 x 3)} = Rs. 7

PROBLEM TYPE-2:

PROBLEM TYPE-2

QUESTIONS BASED ON MIXTURES

Example: A vessel has 60 L of solution of acid and water having 80% acid. How much water is to be added to make a solution in which acid forms 60%?

(a) 48 L

(b) 20 L

(c) 36 L

(d) None of these

Solution: (b)

Given, percentage of acid = 80%

Then, percentage of water = 20%

In 60L of solution, water = (60 x 20)/ 100 = 12L

Let p liter of water be added.

According to the question, => {(12 + p)/ (60 + p)} x 100 = 40 (∵ 100 – 60 = 40% water)

=>1200 + 100p = 2400 + 40p

⇒ 60p = 1200

p= 20L

PROBLEM TYPE-3:

QUESTIONS BASED ON RATIOS AND FRACTIONS

Example: If the numerator of a fraction is increased by 20% and the denominator is decreased by 5%, the value of the new fraction becomes 5/2. The original fraction is:

(a)24/19

(b)3/18

(c)95/48

(d)48/95

Solution: (c)

Let original fraction be p/y

According to the question, {(120/100) p/ (95/100) y} = 5/2

120p/95y = 5/2 => p/y = (5/2) x (95/120) = 95/48.

PROBLEM TYPE-4:

QUESTIONS BASED ON INCOME, SALARY, EXPENDITURE

Example: The monthly income of a person was Rs 13500 and his monthly expenditure was Rs 9000. Next year's income increased by 14% and his expenditure increased by 7%. The percent increase in his savings was:

(a) 7%

(b) 21%

(c) 28%

(d) 35%

Solution: (c)

Given, monthly income = 13500 and expenditure = 9000

Then, original savings= Rs. (13500-9000) = Rs 4500

New income = 114% of Rs. 13500 = Rs 15390

New expenditure= 107% of Rs 9000 = Rs 9630

New saving = Rs. (15390 – 9630) = Rs 5760

NS = new savings, OS = Original savings

Percentage increase in savings = {(NS – OS)/OS} X 100

{(5760 – 4500)/4500} X 100 = (1260/4500) X 100 = 28%

Practice Questions:

Q1. A man distributes 10%, 18% and 22% of his salary into his three children who spend 40%, 60% and 25% of that amount respectively. The difference between the total amount left with the children and man is Rs. 1015. What is the salary of the man?

A. Rs. 6000

B. Rs. 4200

C. Rs. 4800

D. Rs. 5000

E. Rs. 5600

Ans- (d)

Q2. Salary of A is 37.5% of the total salary of A and B. B saves 60% of his salary and total savings of A and B is 50% of their total income. Their average expenditure is Rs 16000. What is the total salary of A and B?

A. Rs. 96000

B. Rs. 54000

C. Rs. 72000

D. Rs. 64000

E. Rs. 48000

Ans- (D)

Q3. In a class 25% of the students passed in both English and Hindi. 37.5% of the students failed in both the subjects while 60% students failed in Hindi. The difference between the students who passed in English and those who passed in Hindi is 15. What is the total number of students in class?

A. 180

B. 420

C. 360

D. 200

E. 240

Ans- (D)

Q4. Out of total students 100/3 % are in hostel A and remaining are in hostel B. If 20 students from hostel B are shifted to hostel A, then total students in hostel A becomes 50% of total students. If 20 students from hostel A are shifted to hostel B, then the total students in hostel A becomes what percent of total students?

A. 26.34%

B. 16.67%

C. 12.75%

D. 20.67

E. None of these

Ans-(B)

Q5. AB de Villiers smashes 86 runs against Australia in 16 balls. If he only scored in boundaries (fours and sixes) only, then find the maximum percent of runs he scored by hitting fours.

A.23.25%

B. 26.4%

C. 74.5%

D. 28%

E. None of these

Ans-(A)

Q6. On a Big Billion-day sale, Google flagship mobile phone was available at a discount of 20% on Flipkart. The customers who are purchasing for the first time on Flipkart will get additional cashback of 10 % on the billing amount. Suraj being the 1st time user of Flipkart purchases the mobile phone for Rs. 36000, finds the actual cost price of the mobile phone.

A. Rs. 50000

B. Rs. 45000

C. Rs. 52250

D. Rs. 47250

E. None of these

Ans- (A)

Q7. As per a company policy only 25% of the female employees and 20% of the male employees can hold the positions higher than level 2. If the ratio of female and male employees in the company is 3: 2, then find the percentage of employees which are working below level 2.

A. 75%

B. 77%

C. 70%

D. 72%

E. 79%

Ans- (B)

Q8. A dishonest salesman buys x% more grains than what he pays for, while selling he uses counterfeit weight which measures 800 grams for every 1000 grams. If he sells the item at 10% above the cost price and earn an overall profit of 65%, then find the value of x.

A. 20%

B.25%

C. 35%

D. 15%

E. None of these

Ans- (A)

Q9. In an exam minimum qualifying marks for class IX and X are 30% and 45% respectively. It is known that total marks of each class are the same and a boy of class X scored 1225; thereby failing by 125 marks. Find passing marks for class IX.

A. 900

B. 1200

C. 1500

D. 925

E. None of these

Ans-(A)

Q10. ABC publication started with 2000 novels. The printing cost, packaging cost and delivery cost of each novel is Rs. 150, Rs. 20 and Rs. 50 respectively. If 40% of the novels are sold at 3/4th of the cost price, then how much percent above the cost price should the remaining novels be sold to get 20% profit on total expenditure?

A. 25%

B. 20%

C. 30%

D. 40%

E. 50%

Ans-(E)

Q11. A pickpocket stole the wallet of Mr. Jittu. Jittu remembers that before he lost his wallet, he bought a notebook and a marker. He pays 1/5th of his money for buying the notebook, and of the remaining, he spends 25% on buying marker which is equal to Rs. 12. Find the amount of money lost by Mr. Jittu.

A. Rs. 125

B. Rs. 75

C. Rs. 100

D. Rs. 60

E. None of these

Ans-(E)

Q12. A survey was conducted in a village to know the reason of Deaths due to Critical Diseases. Number of people who died due to Diabetes were 20% of the total population. It was found that 2000 people died due to lung cancer. The people who died of Diabetes were 1200 more than those who died of Lung Cancer. If the people who died of lung cancer were 33.33 % of the people who smoke, then what percent of the total population were smokers?

A. 40%

B. 62.5%

C. 37.5%

D. 28.50%

E. 32.50%

Ans- (C)

Q13. Rakul spent 10% of his yearly income on house rent, 14% on buying a new car, 12% on kids' school. He spent 15% and 10% of the remaining on groceries and vacation in Spain. If he saved Rs.518400 in the entire year, then find his monthly salary?

A. Rs. 90000

B. Rs. 108000

C. Rs. 98000

D. Rs. 136000

E. None of these

Ans-(A)

Q14. Two villages Rampur and Jamnagar had the same population 2 years ago. Population of Rampur decreased at R% p.a. while the population of Jamnagar increased at R% p.a. Today, the difference between their population is 1000R, then what was the population of any village 2 years ago?

A. 15000

B. 20000

C. 25000

D. Data insufficient

E. None of these

Ans- (C)

Q15. The bank deposit of Rama is 100% more than that of Ajay and 75% more than that of Jatin. Rama's deposits are what percent of the total deposits of Ajay and Jatin together?

A. 93.67%

B. 92.67%

C. 93.33%s

D. 91.33%s

E. None of these

Ans- (C)

Q16. In a school, 40% of students are in high school or above and rest are in junior high school or below. Of those who are in high school or above, the ratio of boys to girls is 7 : 3, and those in junior high school or below have boys to girls in ratio 7 : 5. Ratio of boys in high school or above to junior high school or below:

A. 2: 3

B. 4: 3

C. 3: 4

D. 4: 5

E. None of these

Ans- (D)

Q17. In an examination of SBI SO, Ramu scored 92% marks, Naveen scored 56% and Samarth scored 634 marks out of the total marks. Average marks scored by them was 643. What percentage of the total marks did Samarth get in the SBI SO exam?

A. 66.23%

B. 68.34%

C. 72.45%

D. 76.67%

E. None of these

Ans- (C)

Q18. The speed ratio of A, B and C is 5: 4: 3. All of them start running together on a track and match their respective wrist watches when they finish the race. C completes the race in 20 min. When B finishes the race the wrist watch of A shows 7:27PM. When C finishes the race, his watch shows 7:30PM and wrist watch of B shows 7:16PM. At the start of the race what is the difference between the time in the wrist watch of A and B?

A. 15 min

B. 16 min

C. 12 min

D. 6 min

E. None of these

Ans- (B)

TRIGONOMETRY

Trigonometry is made of three words “tri”, “gono”, “metry”. Where “tri” means “three”, “gono” means “side” and “metry” means measurement. So, trigonometry is study of measuring three side figure which is triangle. $\theta \; \frac{Base}{Hypotenuse}$

Usually we use right angle triangle to solve problem based on trigonometry. Problem in trigonometry are usually based on trigonometric ratio.

Trigonometric Ratio

Trigonometric ratio are the ratio between two sides of a triangle. At particular angle the ratio between two sides will remain same irrespective to their length.

There are six Trigonometric Ratios which are as:

Sine: It is a ratio between a perpendicular and hypotenuse. It is represented as “sin” in all trigonometric identities.

sin $\theta \; \frac{Perpendicular}{Hypotenuse} =$

Where θ represents the angle for which the ratio is derived.

Cosine: It is a ratio between a base and hypotenuse. It is represented as “cos” in all trigonometric identities.

cos $\theta = \frac{Base}{Hypotenuse}$

Secant: It is a ratio between a hypotenuse and base. It is represented as “sec” in all trigonometric identities.

sec $\theta = \frac{Hypotenuse}{Base}$

Cosecant: It is a ratio between a hypotenuse and perpendicular. It is represented as “cosec” in all trigonometric identities.

cosec $\theta = \frac{Hypotenuse}{perpendicular}$

Tangent: It is a ratio between a perpendicular and base. It is represented as “tan” in all trigonometric identities.

tan $\theta \; \frac{perpendicular}{Base} =$

Cotangent: It is a ratio between a base and perpendicular. It is represented as “cot” in all trigonometric identities.

cot θ $\frac{Base}{Perpendicular}$ =

Angle: When two rays (initial and terminal) meet at a point after rotation in a plane then they are said to have described an angle. In other words we can say, the circular distance between two inclined lines is called angle.

Unit of Angle:

- Degree (°)
- Radian (ᶜ)

Relationship between degree and radian:

π rad = 180°

For below particular angles the value of trigonometric ratios are constant.

	0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$∞	N.D/
cot	N.D/ ∞	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2 ∞	N.D/
cosec	N.D/ ∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Signs of Trigonometric Ratio in quadrants:

1st quadrant: All positive

2nd quadrant: sin and cosec positive

3rd quadrant: tan and cot positive

4th quadrant: cos and sec positive

Relation between Trigonometric Ratios:

sin θ × θ cosec = 1

cos θ × θ sec = 1

tan θ × co θ = 1

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sec\theta}{\text{cosec}\,\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\text{cosec}\,\theta}{\sec\theta}$

Trigonometric Ratios of Allied Angles:

With θ

sin (- θ) = - θ sin

cos (- θ) = θ cos

tan (- θ) = - tan

cot (- θ) = - cot θ

sec (- θ) = sec θ

cosec (- θ) = - cosec θ

With (90° – θ)

sin (90° – θ) = cos θ

cos (90° – θ) = sin θ

tan (90° – θ) = θcot

cot (90° – θ) = tan θ

sec (90° – θ) = cosec θ

cosec (90° – θ) = sec θ

With (90° + θ)

sin (90° + θ) = cos θ

cos (90° + θ = – sin θ

tan (90° + θ) = – cot θ

cot (90° + θ) = – tan θ

sec (90° + θ) = – cosec θ

cosec (90° + θ) = – sec θ

With (180° – θ)

$\sin(180^\circ - \theta) = \sin\theta$

$\cos(180^\circ - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

$\cot(180^\circ - \theta) = -\cot\theta$

$\sec(180^\circ - \theta) = -\sec\theta$

$\mathrm{cosec}\,(180^\circ - \theta = \mathrm{cosec}\,\theta$

With $(180^\circ + \theta)$

$\sin(180^\circ + \theta) = -\sin\theta$

$\cos(180^\circ + \theta) = -\cos\theta$

$\tan(180^\circ + \theta) = \tan\theta$

$\cot(180^\circ + \theta) = \cot\theta$

$\sec(180^\circ + \theta) = -\sec\theta$

$\mathrm{cosec}\,(180^\circ + \theta = -\mathrm{cosec}\,\theta$

With $(270^\circ - \theta$

$\sin(270^\circ - \theta = -\cos\theta$

$\cos(270^\circ - \theta) = -\sin\theta$

$\tan(270^\circ - \theta = \cot\theta$

$\cot(270^\circ - \theta) = \tan\theta$

$\sec(270^\circ - \theta) = -\mathrm{cosec}\,\theta$

$\mathrm{cosec}\,(270^\circ - \theta) = -\sec\theta$

With $(270^\circ + \theta)$

$\sin(270^\circ + \theta) = -\cos\theta$

$\cos(270^\circ + \theta = \sin\theta$

$\tan(270^\circ + \theta) = -\cot\theta$

$\cot(270^\circ + \theta) = -\tan\theta$

$\sec(270^\circ + \theta) = \mathrm{cosec}\,\theta$

$\mathrm{cosec}\,(270^\circ + \theta = -\sec\theta$

With (360° – θ)

$\sin(360^\circ - \theta) = -\sin\theta$

$\cos(360^\circ - \theta) = \cos\theta$

$\tan(360^\circ - \theta) = -\tan\theta$

$\cot(360^\circ - \theta) = -\cot\theta$

$\sec(360^\circ - \theta) = \sec\theta$

$\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$

With (360° + θ)

$\sin(360^\circ + \theta) = \sin\theta$

$\cos(360^\circ + \theta) = \cos\theta$

$\tan(360^\circ + \theta) = \tan\theta$

$\cot(360^\circ + \theta) = \cot\theta$

$\sec(360^\circ + \theta) = \sec\theta$

$\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec}\theta$

Some Useful Identities

1) $\sin^2\theta + \cos^2\theta = 1$
   It can also be expressed as
   $\sin^2\theta = 1 - \cos^2\theta$
   $\cos^2\theta = 1 - \sin^2\theta$

2) $\sec^2\theta - \tan^2\theta = 1$
   It can also be expressed as
   $\sec^2\theta = 1 + \tan^2\theta$
   $\sec^2\theta - 1 = \tan^2\theta$

3) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
   It can also be expressed as
   $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
   $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

4) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

5) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

6) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

7) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

8) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

9) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

10) $2 \sin A \sin B = \cos(A-B) - \sin(A+B)$

11) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

12) $\sin^2 A - \sin^2 B = \sin(A+B) \sin(A-B)$

13) $\cos^2 A - \cos^2 B = \cos(A+B) \cos(A-B)$

14) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$

15) $\tan(A-B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$

16) $\cot(A+B) = \frac{\cot A . \cot B - 1}{\cot A + \cot B}$

17) $\cot(A-B) = \frac{\cot A . \cot B + 1}{\cot B - \cot A}$

18) $\sin 2A = 2 \sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$

19) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - \sin^2 A \quad \frac{1-\tan^2 A}{1+\tan^2 A} \quad =$

20) $\tan 2A = \frac{2 \tan A}{1-\tan^2 A}$

21) $\sin 3A = 3 \sin A - 4 \sin^3 A$

22) $\cos 3A = 4 \cos^3 A - 3 \cos A$

23) $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3\tan^2 A}$

24) $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2}\right) \quad \left(\frac{C-D}{2}\right)$. cos

25) $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2}\right) \quad \left(\frac{C-D}{2}\right)$. sin

26) $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2}\right) \quad \left(\frac{C-D}{2}\right)$. cos

27) $\cos C - \cos D = 2 \sin \left(\frac{C+D}{2}\right) \sin \left(\frac{D-C}{2}\right)$

28) If $4\theta < 60$

i. $\sin\theta. \sin 2\theta . \sin 4\theta = ¼ \sin 3\theta$

ii. $\cos\theta . \cos 2\theta . \cos 4\theta = ¼ \cos 3\theta$

iii. $\tan\theta. \tan 2\theta . \tan 4\theta = \tan 3\theta$

iv. $\cot\theta. \cot 2\theta. \cot 4\theta = \cot 3\theta$

29) For all value of θ

i. $\sin (60 - \theta) \sin\theta. \sin (60 + \theta) = \frac{1}{4} \sin 3\theta$

ii. $\cos (60 - \theta) \cos\theta. \cos (60 + \theta = \frac{1}{4} \cos 3\theta$

iii. $\tan (60 - \theta) \tan \theta. \tan (60 + \theta) = \tan 3\theta$

iv. $\cot (60 - \theta) \cot . \cot (60 + \theta) = \cot 3\theta$

30) If $A + B = 45^\circ$

i. $(1 + \tan A) (1 + \tan B) = 2$

ii. $(1 - \cot A) (1 - \cot B) = 2$

31) If $A + B = 90^\circ$

i. $\sin A = \cos B$

ii. $\operatorname{cosec} A = \sec B$

iii. $\tan A = \cot B$

32) If $A + B + C = 90^\circ$

i. $\tan A .\tan B + \tan B.\tan C + \tan C. \tan A = 1$

ii. $\cot A + \cot B + \cot C = \cot A. \cot B. \cot C$

33) If $A + B + C = 180^\circ$

i. $\tan A + \tan B +\tan C= \tan A. \tan B. \tan C$

ii. $\cot A. \cot B + \cot B.\cot C + \cot C. \cot A = 1$

iii. $\sin 2A + \sin 2B + \sin 2C = 4\sin A.\sin B.\sin C$

34) $\tan (45 + \theta) = \frac{1+\tan\theta}{1-\tan\theta} \quad \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} \quad =$

35) tan (45 –) θ $\frac{1-\tan\theta}{1+\tan\theta}$ $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$ =

Solved Examples:

1. If 12tan θ5, then find the trigonometric ratio.

Solution:

tan θ $\frac{5}{12} = \frac{perpendicular}{base}$

It means perpendicular is 5 and base will be 12. By using Pythagoras Theorem, we can easily find hypotenuse.

Hypotenuse^2 = perpendicular^2 + base^2

Hypotenuse = $\sqrt{5^2+12^2}$ = 13

$\frac{1+\sin\theta}{1-\sin\theta}$ $\frac{1+\frac{15}{17}}{1-\frac{15}{17}}$

sin θ $\frac{Perpendicular}{Hypotenuse}$ = $\frac{5}{13}$

cos θ $\frac{Base}{Hypotenuse}$ $\frac{12}{13}$ =

sec θ $\frac{Hypotenuse}{Base}$ $\frac{13}{12}$ =

cosec θ $\frac{Hypotenuse}{perpendicular}$ $\frac{13}{5}$ =

cot θ $\frac{Base}{Perpendicular}$ $\frac{12}{5}$ =

2. If tan = θ, then find the value of $\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta}$ $\frac{a}{b}$

Solution:

$\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta}$

Divide both numerator and denominator by

$\frac{a\frac{\sin\theta}{\cos\theta}+b}{a\frac{\sin\theta}{\cos\theta}-b}$ θ

θ $\frac{\sin\theta}{\cos\theta}$ $\frac{a}{b}$ tan = =

$\frac{a\times\frac{a}{b}+b}{a\times\frac{a}{b}-b} = \frac{a^2+b^2}{a^2-b^2}$

OR

As both numerator and denominator have sin and cos, which have hypotenuse their denominator thus we can use a as θn and b as cc θ

Now,

$\frac{a\sin\theta+b\cos\theta}{a\sin\theta-b\cos\theta} = \frac{a\times a+b\times b}{a\times a-b\times b} = \frac{a^2+b^2}{a^2-b^2}$

$\theta \quad \frac{15}{8}$

3. sin 720° – cot 270° – sin 150° cos 120° is equal to –

Solution:

sin (2×360° + 0°) – cot (360° – 90°) – sin (180° – 30°). cos (90° + 30°)

sin 0° – cot 90° – sin 30°. sin 30°

$0 - 0 + \quad \frac{1}{2} \times \frac{1}{2} = \quad \frac{1}{4}$

4. If tan (x + y) . tan (x – y) = 1, then find the value of tan x

Solution:

tan (x + y) = $\frac{1}{\tan(x-y)}$: cot (x – y)

x + y + x – y = 90°

2x = 90°

x = 45°

tan 45° = 1

5. If cot 2A cot 3A = 1 , then find the value of sin $\frac{5A}{2}$. cos $\frac{5A}{2}$

Solution:

2A + 3A = 90°

A = 18°

Sin $\frac{5\times18}{2}$ cos $\frac{5\times18}{2}$

sin 45° × cos 45° = $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

6. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{17}{7}$ then find the value of tan .θ

Solution:

Apply componendo and dividendo –

$$\frac{\sin\theta + \cos\theta + \sin\theta - \cos\theta}{\sin\theta + \cos\theta - \sin\theta + \cos\theta} = \frac{17+7}{17-7}$$

$\frac{2\sin\theta}{2\cos\theta}$ = tan= θ $\frac{24}{10}$ = $\frac{12}{5}$

7. If $\tan\theta + \cot\theta = 2$, then find the value of $\tan^{100}\theta + \cot^{110}\theta$

Solution:

$\tan^{100}\theta + \cot^{110}\theta = 1 + 1 = 2$

8. If $\sin^2\theta + \sin\theta = 1$, then find the value of $\cos^4\theta + \cos^2\theta$.

Solution:

$\sin\theta = 1 - \sin^2\theta = \cos^2\theta$

$\cos^4\theta = \sin^2\theta$

$\cos^4\theta + \cos^2\theta = \sin^2\theta + \sin\theta = 1$

9. Solve: tan1°.tan2°.tan3° … tan89°

Solution:

tan1°.tan2°.tan3° … tan45° … cot 3°.cot 2°.cot 1°

tan1° cot 1° tan2° cot 2° tan3° cot 3°… tan45° 1×1×1….×1 = 1

10. Solve cos1°. cos2°. cos3°…… cos179°.

Solution:

cos1°. cos2°. cos3°… cos 90° … cos179°

cos 90° = 0

cos1°. cos2°. cos3°…0… cos179° = 0

MENSURATION

Mensuration is mainly divided into three parts:

One is **Area**, second is **Volume** and third is **Perimeter.**

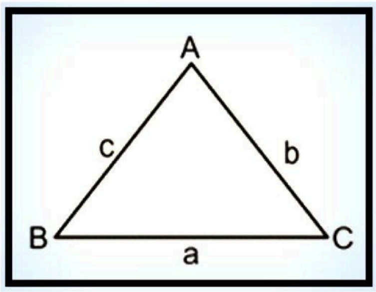
Triangle

Perimeter = a + b + c

Area of Triangle = [½ × base × height] or

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Here, s is a semi perimeter



Q1. Sides of a triangle are 12m, 13m and 11m. Find the area of triangle and height with respect to the side of 12 cm.

Solution:

S= $\frac{12+13+11}{2} = \frac{36}{2} = 18m$

Area of triangle =$\sqrt{s(s-a)(s-b)(s-c)}$

Area= $\sqrt{18\times6\times5\times6}$

Area= $6\sqrt{105}\ m^2$

Side=12m (Base), height=?

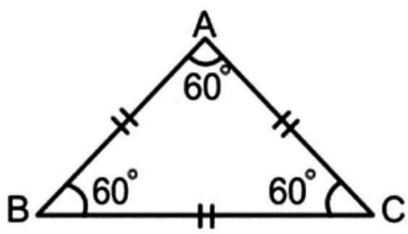
Area of triangle =1/2×base×Height

$6\sqrt{105}$ = ½ × 12 × height

Height = $\sqrt{105}$ m

Properties of Equilateral Triangle

- Properties of equilateral triangle
- All three sides are equal.
- All three angles are 60°



Formulae

- Perimeter = 3 × side = 3a
- Area of equilateral triangle = $(\sqrt{3}/4)a^2$
- Height = Median = $(\sqrt{3}/2)a$
- Side of an equilateral triangle = $\frac{\text{The sum of distances of a point inside a triangle from all the sides} \times 2}{\sqrt{3}}$

Q2. The area of an equilateral triangle is 49√3 m2, find the value of its height and side?

Solution:

The area of an equilateral triangle = $49\sqrt{3}\ m^2$
$(\sqrt{3}/4)a^2 = 49\sqrt{3}$
$a^2 = 49 \times 4$

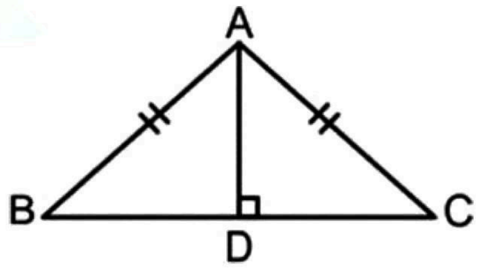
a = 14 m

Height = $(\sqrt{3}/2)a$

Height = $(\sqrt{3}/2) \times 14 = 7\sqrt{3}$ m

Properties of Isosceles Triangle

- Two sides are equal.
- AB = AC



Formula:
Area of Δ = ½ × base × height

Q3. In an isosceles triangle, if the ratio of equal side and unequal side is 5: 8 and its perimeter is 36 cm. Find the area of this triangle.

Solution:

Let the equal and unequal side be 5x and 8x

Perimeter of this triangle = 5x + 5x + 8x

36 = 18x

x = 2

Sides of this triangle are 10 cm, 10 cm and 16 cm

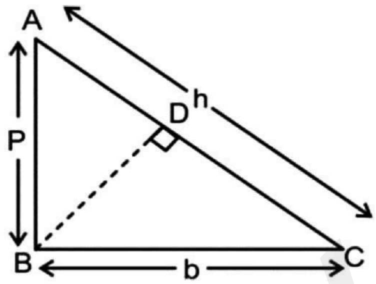
Perpendicular on unequal side = $\sqrt{[10^2 - (16/2)^2]} = \sqrt{[100 - 64]}$

Perpendicular on unequal side = 6 cm

Area of isosceles triangle = ½ × unequal side × perpendicular on unequal side

Area of isosceles triangle = ½ × 16 × 6 = 48 cm^2

Properties of Right-Angle Triangle



- $\text{Hypotenuse}^2 = (\text{base})^2 + (\text{height})^2$
- Area = ½ × base × height = ½ × b × p
- Length of Perpendicular at hypotenuse = $\frac{Height \times Base}{Hypotenuse}$

Q4. The area of a right-angle triangle is 30 m and base is 7 m more than height, find the length of perpendicular on hypotenuse.

Solution:

Let the height of the triangle be x

⇒ Base = x + 7

⇒ Area = ½ × base × height

⇒ 30= ½ × (x + 7) × x

⇒ $60 = x^2 + 7x$

⇒ $x^2 + 7x – 60 = 0$

⇒ (x + 12) (x – 5) = 0

⇒ x = 5 m

⇒ Base = x + 7 = 12 m

⇒ $\text{Hypotenuse}^2 = (\text{base})^2 + (\text{height})^2$

⇒ $\text{Hypotenuse}^2 = 144 + 25 = 169$

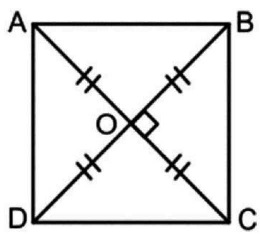
⇒ Hypotenuse = 13 m

The length of perpendicular on hypotenuse is (60/13) m.

Square

Properties of square

- Opposite sides of a square are both parallel and equal in length.
- The diagonal of a square is equal. AC = BD
- The diagonals of a square bisect each other and meet at 90°. AO = OC, BO = OD and ∠BOC = 90°
- All four angles of a square are 90°.



Formula:

- Perimeter = 4 × side
- Area = $(\text{side})^2$ = ½ × (diameter)
- Diameter = √2 side

Q5. Area of a square is 1352 m2. Find the perimeter and the length of the diagonal.

Solution:

Area of a square = 1352

$(\text{Side})^2$ = 1352

Side = 26√2 m

Perimeter = 4 × side

Perimeter = 4 × 26√2 = 104√2 m

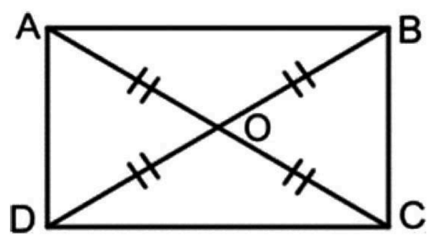
Length of diagonal = √2 × side

Length of diagonal = √2 × 26√2 = 52 m

Rectangle

Properties of rectangle

- Opposite sides of a rectangle are equal in length.
  AB = DC, AD = BC
- Opposite sides of a rectangle are parallel.
  AB || CD, AD || BC
- The diagonals of a rectangle bisect each other.
  AO = OC, BO = OD
- All four angles of a square are 90°.



Formulae:

Area = l × b

Perimeter = 2(l + b)

Diameter = √ $(l^2 + b^2)$

Here, Length = l
Breadth = b

Q6. In a rectangle diagonal is 9 times of its length. Find the ratio of length and breath.

Solution:

Let the length of rectangle be x

Diagonal = 9x

Diameter = $\sqrt{(l^2 + b^2)}$

$9x = \sqrt{(x^2 + b^2)}$

$x^2 + b^2 = 81 x^2$

$80 x^2 = b^2$

$b = (4\sqrt{5})x$

Length: breath = x: $(4\sqrt{5})x$

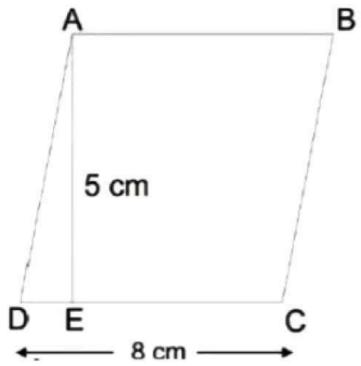
Length: breath = 1: $4\sqrt{5}$

Parallelogram

In a parallelogram, the opposite sides are parallel and equal. But all 4 sides are not equal. Also, the angle of the corners is not at 90 degrees. Whereas in square and rectangle, the angle of corners is 90 degrees.

- Area of parallelogram = l × h
- Perimeter of parallelogram = 2(l+b)

Here, l = length or base of parallelogram. h = height of parallelogram.



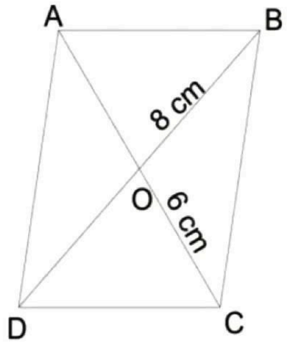
Rhombus

Rhombus is also a parallelogram. The main factor is that all 4 sides are equal in the rhombus. The mensuration formula chart for the rhombus is,

- Area of rhombus = d1 × d2 / 2

- The perimeter of rhombus = 4l

Here, d1 and d2 are the lengths of the diagonals. l is the side of the rhombus.

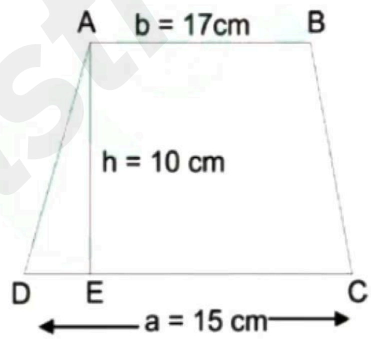


Trapezium

In trapezium, one set of opposite sides are parallel and unequal. The other set of opposite sides are not parallel.

- Area = 1 / 2 h(a+b)
- Perimeter = Sum of all sides

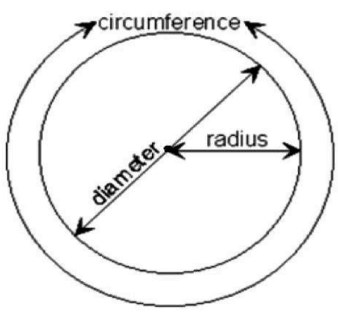
Here a and b are the top and bottom sides of the trapezium. h is the height of the trapezium.



Circle

- Circumference of a circle = π x diameter
- Diameter of circle = 2r
- Area of a circle = π x r x r

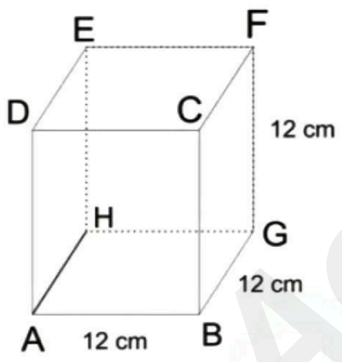
Here r is the radius of the circle.



Cube

Cube is a 3d shape of a square. In the cube all the length, breadth, and height are equal. The mensuration formula for bank exams also includes all 3d shapes. So, refer to the mensuration formulas for the cube here.

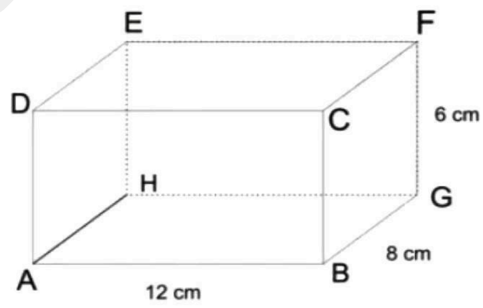
- Volume of a cube = (side)3
- Total surface area of a cube = 6 × (side)2
- Diagonal of cube = √3 x (side)



Cuboid

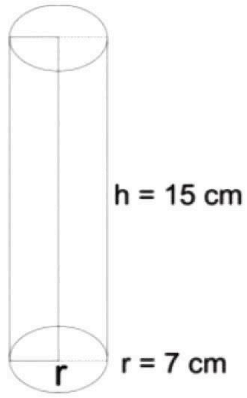
It is a 3d shape of a rectangle. The mensuration formulas for the cuboid are,

- Volume of a cuboid = (length × breadth × height) = lbh
- Total surface area of cuboid = 2(lb + bh + hl)



Cylinder

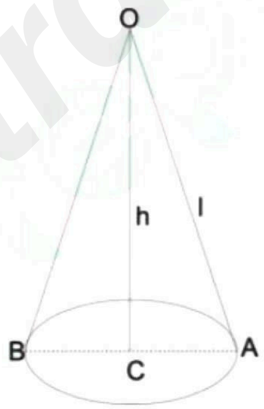
- Area of curved surface = (perimeter of base) x height = $2\pi rh$
- Total surface area = $2\pi r (r + h)$
- Volume = π x r x r x h



Cone

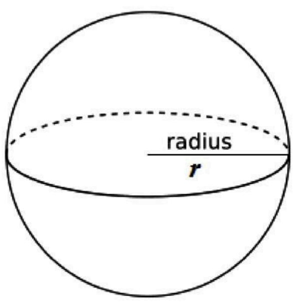
- Curved surface area = πrl
- Total surface area = $\pi r(r + l)$
- Volume of cone = (1/3) x π x r x r x h

Here l is the slant height of the cone.



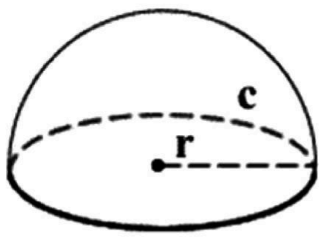
Sphere

- r = radius
- Volume: $V = \frac{4}{3}\pi r^3$
- Curved Surface Area = Total Surface Area = $4\pi^2$



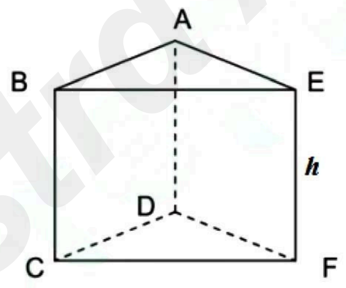
Hemisphere

- Volume = $\frac{2}{3}\pi r^3$
- Curved surface area = $2\pi r^2$
- Total surface area = $3\pi r^2$



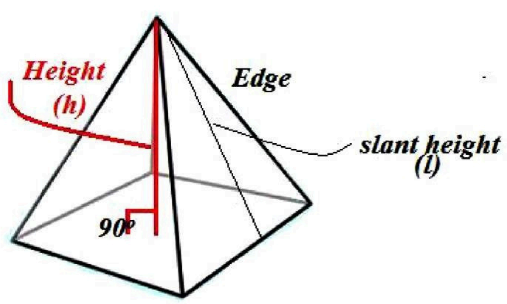
Prism

- Volume = Base area x height
- Lateral Surface area = perimeter of the base x height
- Total Surface area = LSA + 2(Area of base)



Pyramid

- Volume of a right pyramid = $\frac{1}{3}$ X area of the base height X height
- Area of the lateral faces of a right pyramid = $\frac{1}{2}$ x perimeter of the base x slant height
- Area of whole surface of a right pyramid = Area of the lateral faces + Area of the base.



Practice Questions:

Q1. Three cubes of metal whose edges are in ratio of 3:4:5, are melted and one new cube is formed. If the diagonal of the cube is 18√3 cm, then find the edge of the largest among three cubes.

(A) 18 cm
(B) 24 cm
(C) 15 cm
(D) 12 cm

Q2. What is the area of the largest triangle that can be fitted into a rectangle of length 'a' units and width 'b' units?

(A) unit2
(B) unit2
(C) unit2
(D) unit2

Q3. Find the perimeter and area of an isosceles triangle whose equal sides are 5 cm and the height is 4 cm.

(A) 24 cm2 , 13 cm
(B) 18 cm2 , 16 cm
(C) 12 cm2 , 13 cm
(D) 12 cm2 , 16 cm

Q4. A parallelogram has area A sq. mts. A second parallelogram is formed by joining the mid-points of its sides. A third parallelogram is formed by joining the mid-poinds of the sides of the second parallelogram. This process is continued upto infinite. What is the sum of area (in sq. mts) of all the parallelograms so formed?

(A) A
(B) $\frac{3A}{2}$
(C) 2A
(D) $\frac{A}{2}$

Q5. The radius of a cylindrical milk container is half its height and surface area of the inner part is 616 sq.cm. The amount of milk that the container can hold, approximately, is
[Use: √5= 2.23 and p = 22/7]

(A) 1.42 litres
(B) 1.53 litres
(C) 1.71 litres
(D) 1.82 litres

Q6. From the four corners of a rectangular sheet of dimensions 25 cm x 20 cm, square of side 2 cm is cut off from four corners and a box is made. The volume of the box is-

(A) 828 cm3
(B) 672 cm3
(C) 500 cm3

(D) 1000cm3

Q7. A rectangular paper sheet of dimensions 22 cm × 12 cm is folder in the form of a cylinder along its length. What will be the volume of this cylinder?

(A) 460 cm^3
(B) 462 cm^3
(C) 624 cm^3
(D) 400 cm^3

Q8. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diameter and 18 m length. The radius (in cm) of the wire is:

(A) $\frac{2}{15}$

(B) $\frac{1}{15}$

(C) $\frac{1}{30}$

(D) 15

Q9. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8 gm/cm3, then the weight of the pipe is:

(A) 3.696 kg
(B) 3.6 kg
(C) 36 kg
(D) 36.9 kg

Q10. Water flows at the rate of 10 meters per minute from cylindrical pipe 5 mm in diameter how long it will take to fill up a conical versel whose diameter at the base is 30 cm and depth 24 cm?

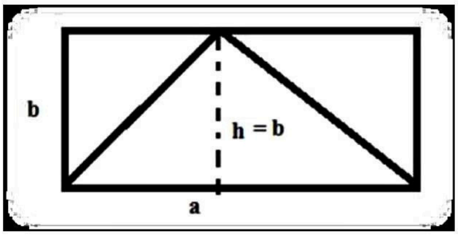
(A) 28 minutes 48 seconds
(B) 51 minutes 12 seconds
(C) 51 minutes 24 seconds
(D) 28 minutes 36 seconds

Solution:

Q1. (C)

Total volume = (3x)3+(4x)3+(5x)3 = 216x3
Then side of new cube/= 6x
Then diagonal of new cube = 6x√3 ⟶ 18√3 x = 3
Then side of larger cube = 5×3 = 15

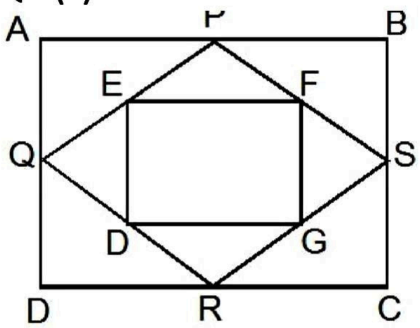
Q2. (B)



Area of triangle = (1/2) base × height
= (1/2) ab unit2

Q3. (D)
12 cm^2, 16cm

Q4. (C)



Sum of areas:

$$= A + \frac{1}{2}A + \frac{1}{4}A + - - - \infty$$

$$= \frac{A}{1 - \frac{1}{2}} = 2A$$

Q5. (B)
Let radius of the container is x cm.
Then height of the container is 2x cm.
According to question 2πr(2r) + πr2 = 616
5πr2 = 616

$$r^2 = \frac{616 \times 7}{5 \times 22}$$

$$r = \frac{14}{\sqrt{5}}$$

$$V = \frac{22}{7} \times \frac{14}{\sqrt{5}} \times \frac{14}{\sqrt{5}} \times \frac{28}{\sqrt{5}}$$

= 1546.90 ml.
= 1.54 lit.

Q6. (B)

Volume of the box = 16×21×2 = 672 cm^3

Q7. (B)

$$2\pi r = 22$$

$$R = \frac{7}{2} \text{ cm}$$

$$v = \pi R^2 h$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$$

$$= 462 \text{ cm}^3$$

Q8. (C)

$$\pi r^2 h = \pi R^2 h = \frac{1}{4} \times 8 = 2$$

$$r^2 = \frac{2}{1800} = \frac{1}{900}$$

$$r = \frac{1}{30} \text{ cm.}$$

Q9. (A)

$$r = \pi\left(R^2 - r^2\right) \times h$$

$$= \frac{22}{7} \times \left(4^2 - 3^2\right) \times 21 = 462 \text{ cm}^3$$

$$\therefore 1\text{cm}^3 = 8\,\text{g}$$

$$462 \text{ cm}^3 = 462 \times 8\text{g}$$

$$= 2696 \text{ g} = 3.696 \text{ kg.}$$

Q10. (A)

Volume of water flowing from the pipe in 1 minute

$\pi \times .25 \times .25 \times 1000$

$r = \frac{1}{3}\pi \times 15 \times 15 \times 24$

$T = \frac{1}{3} \times \frac{\pi \times 15 \times 15 \times 24}{\pi \times .25 \times .25 \times 1000} = 28\frac{4}{5}$

28 minutes 48 seconds

SIMPLE AND COMPOUND INTEREST

Definition

Interest	Interest is the amount to be paid on the borrowed money or the amount received on the money lent
Principal	The borrowed money or the lent money is called Principal.
Amount	The sum of the interest and the principal is called the Amount.
Interest Rate	The rate at which the interest is charged on the principal is called Rate of Interest.
Time	The period for which the money is borrowed or deposited is called Time.

Interest can be classified in two types:

1) Simple Interest
2) Compound Interest

Simple Interest

When the interest is calculated only on the Principal for every year, it is called Simple Interest.

Simple Interest can be calculated by the formula:

SI= $\frac{P \times R \times T}{100}$

Where, P = Principal, r = Rate of interest per year, t = Time period in years

Points to Remember

- When the time period is given in months, we convert it into year by dividing it by 12.
- When the time period is given in days, we convert it into year by dividing it by 365.

Q1. Rs.1080 invested for 3 months gave an interest of Rs.27. The simple rate of interest per annum was:

Solution: 3 months

$=\frac{3}{12}$years

SI = $\frac{PRT}{100}$

⇒ 27 = (1080 × r × 3/12)/100
⇒ 27 = (90 × r × 3)/100
⇒ 27 = 270r/100
∴ r = 10%.

Compound Interest

It is the interest paid on the original principal amount and the accumulated past interest.

Formulas related to compound Interest:

A= P $\left(1 + \frac{R}{100}\right)^T$

CI = A - P

Points to Remember

- When rate is compounded half yearly, then we take rate half and time double, then A=P $\left(1 + \frac{\left(\frac{R}{2}\right)}{100}\right)^{2T}$
- When rate is compounded quarterly, then we take rate one fourth and time 4 times, Then A= P$\left(1 + \frac{\left(\frac{R}{4}\right)}{100}\right)^{4T}$
- If rate of compound interest differs from year to year, then A= P $\left(1+\frac{R1}{100}\right)\left(1 + \frac{R2}{100}\right)\left(1 + \frac{R3}{100}\right)$

Q2. Find compound interest on Rs.50000 at 12% per annum for 1 year if compounded half yearly.

Solution:

Amount = P $\left(1 + \frac{\frac{r}{2}}{100}\right)^{2t}$

$=50000\left(1 + \frac{\frac{12}{2}}{100}\right)^{2\times1}$

$=50000(\frac{106}{100})^2$

= Rs 56180

∴ C.I. = A – P = 56180 – 50000 = Rs 6180.

Q3. What will be the amount if a sum of Rs.25000 is placed at CI for 3 years while rate of interest for the first, second, and third years is 4%, 8%, and 10%, respectively?

Solution.

A = P (1 + r_1/100) (1 + r_2/100) (1 + r_3/100)

= 25000 (1 + 4/100) (1 + 8/100) (1 + 10/100)

= 25000 (104/100) (108/100) (110/100) = 30888.

Tree-method

In this method we assume principle (on the basis of rate and time given) such that it eases our calculation part and at the end we compare it to the value given in question to get the required answer.

For example – If 10% interest rate is given for 2 years then we will assume principle as Rs. 100 and if times is 3 years, then we will assume principle as Rs. 1000. It is done to avoid any calculation in decimal form.

Q4. Find compound interest for principal Rs 10000, time = 3 years and rate = 10%.

Solution.

Normal Method:

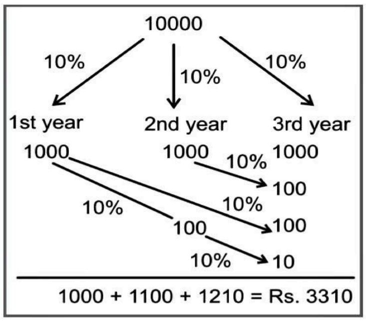
Amount= $P(1 + \frac{r}{100})^t$

= $10000 (1 + \frac{10}{100})^3$

=Rs 13310

C.I. = A − P = 10000 − 13310 = Rs. 3310

Tree Method:



Step 1: Take principle (Rs 10000 here).

Step 2: For year at 10%, interest =

Rs. 1000

For 2^{nd} year, total interest =

interest on principle +

interest on interest of 1^{st} year =

1000 + 100 = Rs. 1100

For 3^{rd} year, total interest =

interest on principle +

interest on interest of 1^{st} year +

interest on interest of 2^{nd} year =

1000 + 100 + 100 + 10 = Rs. 1210

Step 3: Add all interests = 1000 + 1100 + 1210 = Rs. 3310.

Effective Rate Method

Effective rate for 3 years at rate of 10% = $3a.3a^2a^3$ = 33.1 %
Hence, compound interest = 10000 × 33.1% = Rs. 3310.

Q5. Rs.9200 is invested at compound interest at the rate of 25% per annum for 2 years.

Solution.

Normal Method:

Compound interest = $P \times [(1 + r/100)^t - 1]$

Compound interest earned = $9200 \times [(1 + 25/100)^2 - 1] = 9200 \times 0.5625 =$ Rs. 5175

Effective Rate method:

Effective rate = $x + y + xy/100 = 25 + 25 + (25 \times 25)/100 = 56.25\%$

Hence, C.I. = $9200 \times 56.25\%$ = Rs 5175

Q6. What will be the difference between the compound interest and simple interest on Rs.3000 at 10% rate of interest for 2 years?

Solution.

$D = PR^2/10^4$

$\Rightarrow D = (3000 \times 10^2)/10000$

$\therefore \quad D =$ Rs.30

Installment

Q7. The oven set is bought on 4 yearly installments at 10% simple interest. If equal instalments of Rs.2500 are made then find the amount to be paid as the price of the oven set.

Solution:

Given rate = 10% and time = 4 years

Let installment be Rs 100

Then, price = 1st payment + 2nd payment + 3rd payment + 4th payment = 100 + 110 + 120 + 130 = 460

Comparing with given installment, we get price, P = $2500 \times 460/100$ = Rs.11500

LOGARITHMS

- A logarithm of a number with a base is equal to another number. A logarithm is just the opposite function of exponentiation. For example, if $10^2 = 100$ then $\log_{10} 100 = 2$.
- Hence, we can conclude that,

  $\text{Log}_b\, x = n$ or $b^n = x$

  Where b is the base of the logarithmic function.

- A logarithm is defined as the power to which number must be raised to get some other values. It is the most convenient way to express large numbers. A logarithm has various important properties that prove multiplication and division of logarithms can also be written in the form of logarithm of addition and subtraction.
- The logarithm of a positive real number a with respect to base b, a positive real number not equal to 1[nb 1], is the exponent by which b must be raised to yield a.

  i.e $b^y = a$ and it is read as "the logarithm of a to base b."

- In other words, the logarithm gives the answer to the question "How many times a number is multiplied to get the other number?".

Example: How many 3's are multiplied to get the answer 27?

If we multiply 3 for 3 times, we get the answer 27.
Therefore, the logarithm is 3.
The logarithm form is written as follows:
$\text{Log}_3 (27) = 3$ ….(1)
Therefore, the base 3 logarithm of 27 is 3.
The above logarithm form can also be written as:
3x3x3 = 27
$3^3 = 27$ …..(2)
Thus, the equations (1) and (2) both represent the same meaning.

Logarithm Types

In most cases, we always deal with two different types of logarithms, namely

- Common Logarithm
- Natural Logarithm

Common Logarithm

The common logarithm is also called the base 10 logarithms. It is represented as log10 or simply log. For example, the common logarithm of 1000 is written as a log (1000). The common logarithm defines how many times we have to multiply the number 10, to get the required output.

For example, log (100) = 2

If we multiply the number 10 twice, we get the result 100.

Natural Logarithm

The natural logarithm is called the base e logarithm. The natural logarithm is represented as ln or loge. Here, "e" represents the Euler's constant which is approximately equal to 2.71828. For example, the natural logarithm of 78 is written as ln 78. The natural logarithm defines how many we have to multiply "e" to get the required output.

For example, ln (78) = 4.357.

Thus, the base e logarithm of 78 is equal to 4.357.

Logarithm Rules and Properties

There are certain rules based on which logarithmic operations can be performed. The names of these rules are:

- Product rule
- Division rule
- Power rule/Exponential Rule
- Change of base rule
- Base switch rule
- Derivative of log
- Integral of log

Let us have a look at each of these properties one by one

Product Rule

In this rule, the multiplication of two logarithmic values is equal to the addition of their individual logarithms.

$\text{Log}_b (mn) = \log_b m + \log_b n$

For example: $\log_3 (2y) = \log_3 (2) + \log_3 (y)$

Division Rule

The division of two logarithmic values is equal to the difference of each logarithm.

$\text{Log}_b (m/n) = \log_b m - \log_b n$

For example, $\log_3 (2/ y) = \log_3 (2) - \log_3 (y)$

Exponential Rule

In the exponential rule, the logarithm of m with a rational exponent is equal to the exponent times its logarithm.

$\text{Log}_b (m^n) = n \log_b m$

For example: $\log_b(2^3) = 3 \log_b 2$

Change of Base Rule

$\log_b m = \log_a m / \log_a b$

For example: $\log_b 2 = \log_a 2/\log_a b$

Base Switch Rule

$\log_b (a) = 1 / \log_a (b)$

For example: $\log_b 8 = 1/\log_8 b$

Derivative of log

If $f(x) = \log_b(x)$, then the derivative of f(x) is given by;

$f'(x) = 1/(x \ln(b))$

For example: Given, $f(x) = \log_{10}(x)$

Then, $f'(x) = 1/(x \ln(10))$

Integral of Log

$\int \log_b(x)dx = x(\log_b(x) – 1/\ln(b)) + C$

Example: $\int \log_{10}(x)\, dx = x \cdot (\log_{10}(x) – 1 / \ln(10)) + C$

Other Properties

Some other properties of logarithmic functions are:

- $\log_b b = 1$
- $\log_b 1 = 0$
- $\log_b 0$ = undefined

Logarithmic Formulas

$\log_b(mn) = \log_b(m) + \log_b(n)$

$\log_b(m/n) = \log_b (m) – \log_b (n)$

$\log_b (xy) = y \log_b(x)$

$\log_b m\sqrt{n} = \log_b n/m$

$m \log_b(x) + n \log_b(y) = \log_b(x^m y^n)$

$\log_b(m+n) = \log_b m + \log_b(1+nm)$

$\log_b(m – n) = \log_b m + \log_b (1-n/m)$

Solved Examples

Question 1: Solve $\log_2 (64)$ =?

Solution:

since $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, 6 is the exponent value and $\log_2 (64) = 6$.

Question 2: What is the value of $\log_{10}(100)$?

Solution: In this case, 10^2 yields you 100. So, 2 is the exponent value, and the value of $\log_{10}(100) = 2$

Question 3: Use of the property of logarithms, solve for the value of x for $\log_3 x = \log_3 4 + \log_3 7$

Solution: By the addition rule, $\log_3 4 + \log_3 7 = \log_3 (4 * 7)$

$\text{Log}_3 (28)$. Thus, x= 28.

Question 4: Solve for x in $\log_2 x = 5$

Solution: This logarithmic function can be written In the exponential form as $2^5 = x$

Therefore, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, X= 32.

Logarithmic Function Definition

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For x, a > 0, and a≠1,

$y = \log_a x$, if $x = a^y$

Then the logarithmic function is written as:

$f(x) = \log_a x$

The most common bases used in logarithmic functions are base e and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by $\log_{10}$ or simply log.

$f(x) = \log_{10}$

The log function to the base e is called the natural logarithmic function and it is denoted by $\log_e$.

$f(x) = \log_e x$

To find the logarithm of a number, we can use the logarithm table instead of using a mere calculation. Before finding the logarithm of a number, we should know about the characteristic part and mantissa part of a given number:

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

Step 1: Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.

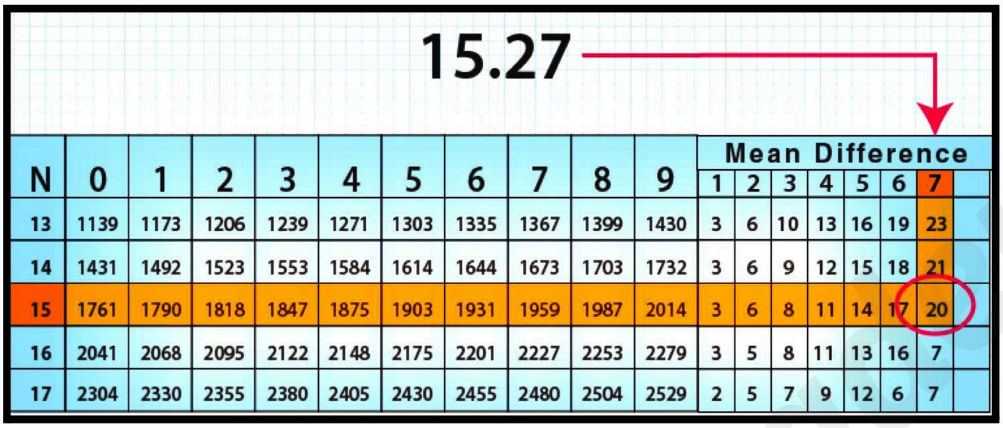
Step 2: Identify the characteristic part and mantissa part of the given number. For example, if you want to find the value of $\log_{10}(15.27)$, first separate the characteristic part and the mantissa part.

Characteristic Part = 15

Mantissa part = 27

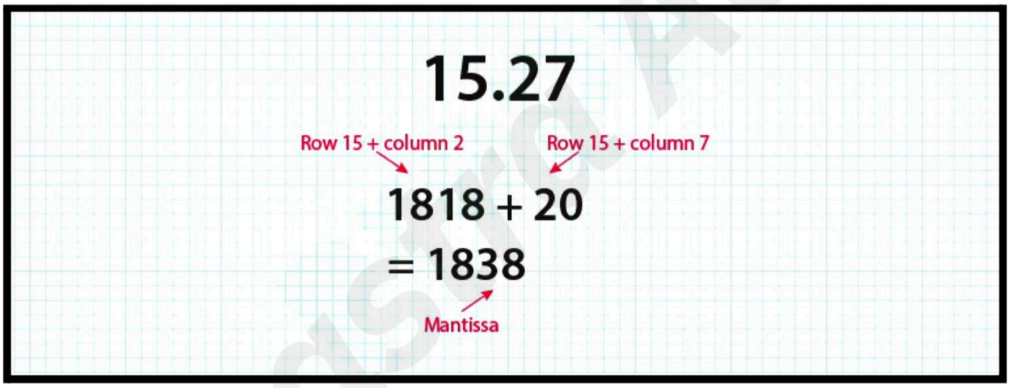
Step 3: Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.

Step 4: Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.



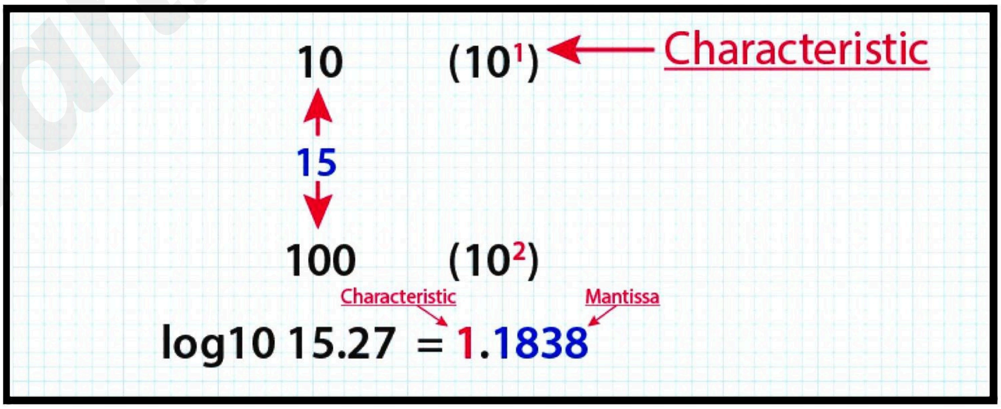
N	0	1	2	3	4	5	6	7	8	9	Mean Difference						
											1	2	3	4	5	6	7
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23
14	1431	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	7
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	9	12	6	7

Step 5: Add both the values obtained in step 3 and step 4. That is 1818+20= 1838. Therefore, the value 1838 is the mantissa part.



Step 6: Find the characteristic part. Since the number lies between 10 and 100, (10^1 and 10^2), the characteristic part should be 1.

Step 7: Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.



Example: Find the value of $\log_{10}$ 2.872

Solution:

Step 1: Characteristic Part= 2 and mantissa part= 872

Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.

Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3

Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.

Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So, the characteristic part is 0

Step 6: Finally combine the characteristic part and the mantissa part. So, it becomes 0.4582.

Therefore, the value of log 2.872 is 0.4582.

Practice Questions

1. Express $5^3 = 125$ in logarithm form.

Solution:

$5^3 = 125$

As we know,

$a^b = c \Rightarrow \log_a c = b$

Therefore;

$\text{Log}_5 125 = 3$

2. Express $\log_{10} 1 = 0$ in exponential form.

Solution:

Given, $\log_{10} 1 = 0$

By the rule, we know;

$\log_a c = b \Rightarrow a^b = c$

Hence,

$10^0 = 1$

3. Find the log of 32 to the base 4.

Solution: $\log_4 32 = x$

$4^x = 32$

$(2^2)^x$ = 2x2x2x2x2

$2^{2x} = 2^5$

2x=5

x=5/2

Therefore,

$\log_4 32$ =5/2

4. Find x if $\log_5(x-7)=1$.

Solution: Given,

$\log_5(x-7)=1$

Using logarithm rules, we can write;

5^1 = x-7

5 = x-7

x=5+7

x=12

5. If $\log_a m=n$, express a^{n-1} in terms of a and m.

Solution:

$\log_a m=n$

$a^n=m$

$a^n/a=m/a$

$a^{n-1}=m/a$

6. Solve for x if $\log(x-1)+\log(x+1)=\log_2 1$

Solution: $\log(x-1)+\log(x+1)=\log_2 1$

log(x-1)+log(x+1)=0

log[(x-1)(x+1)]=0

Since, log 1 = 0

(x-1)(x+1) = 1

$x^2-1=1$

$x^2=2$

$x = \pm \sqrt{2}$

Since, log of negative number is not defined.

Therefore, $x = \sqrt{2}$

7. Express log(75/16) - 2log(5/9) + log(32/243) in terms of log 2 and log 3.

Solution: log(75/16) - 2log(5/9) + log(32/243)

Since, $n\log_a m = \log_a m^n$

⇒ $\log(75/16) - \log(5/9)^2 + \log(32/243)$

⇒ log(75/16) - log(25/81) + log(32/243)

Since, $\log_a m - \log_a n = \log_a(m/n)$

⇒ log[(75/16) ÷ (25/81)] + log(32/243)

⇒ log[(75/16) × (81/25)] + log(32/243)

⇒ log(243/16) + log(32/243)

Since, $\log_a m + \log_a n = \log_a mn$

⇒ log(32/16)

⇒ log2

8. Express 2logx + 3logy = log a in logarithm free form.

Solution: 2logx + 3logy = log a

$\log x^2 + \log y^3 = \log a$

$\log x^2y^3 = \log a$

$x^2y^3 = \log a$

9. Prove that: 2log(15/18) - log(25/162) + log(4/9) = log2

Solution: 2log(15/18) - log(25/162) + log(4/9) = log2

Taking L.H.S.:

⇒ 2log(15/18) - log(25/162) + log(4/9)

⇒ $\log(15/18)^2 - \log(25/162) + \log(4/9)$

⇒ log(225/324) - log(25/162) + log(4/9)

⇒ log[(225/324)(4/9)] - log(25/162)

⇒ log[(225/324)(4/9)]/(25/162)

$\Rightarrow \log(72/36)$

$\Rightarrow \log 2$ (R.H.S)

10. Express $\log_{10}(2+1)$ in the form of $\log_{10}x$.

Solution: $\log_{10}(2+1)$

$= \log_{10}2 + \log_{10}1$

$= \log_{10}(2\times10)$

$= \log_{10}20$

ALGEBRIC EQUATIONS

Algebraic equations are polynomial equations. In examination, generally equations of 1 degree, 2 degree or 3 degrees are asked.

Linear Equation

Polynomial equations with degree 1 i.e., ax + c = 0 are called as linear equations. Some examples of linear equations are as follows –

$2x + 3y = 4$

$x + y + z = 10$

Q. In this question two equations numbered I and II are given. You have to solve both the equations and find out the relation between x and y.

I. 5x = 7y + 21

II. 11x + 4y + 109 = 0

Solution:

I. $2x + 3y = 13$ (1)

II. $3x + 2y = 12$ (2)

(3 × Equation 2) - (2 × Equation 1) gives us

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

Putting value of x in equation 1, we get y

= 3

Hence, $x < y$.

Q. In the given question, two equations numbered l and II are given. You have to solve both the equations and mark the appropriate answer:

I. 4x + 5y = 14

II. 2x + 3y = 5

Solution:

$4x + 5y = 14$ (1)

$2x + 3y = 5$ (2)

On multiplying equation (2) by 2

$4x + 6y = 10$ (3)

Subtracting equation (1) from equation (3),

$y = -4$

$x = 1$ (on putting value of y in the above equation)

$\therefore x > y$.

Quadratic Equation

Polynomial equations with degree 2 i.e., $ax^2 + bx + c = 0$ are called quadratic equations. Some examples of quadratic equations are as follows –

$x^2 + 2x + 3 = 0$

$y^2 - 3y + 4 = 0$

Methods to solve quadratic equation

1) Factorization method

In this quadratic equation $ax^2 + bx + c = 0$ is factorized as $(x - \alpha)(x - \beta) = 0$ and then equation is solved to get $x = \alpha$ or $x = \beta$.

Q. Solve quadratic equation

$x^2 - 2x - 15 = 0$

Solution:

$x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x - 5) + 3(x - 5) = 0$

$\Rightarrow (x + 3)(x - 5) = 0$

$\Rightarrow x + 3 = 0$ or $x - 5 = 0$

$\Rightarrow x = -3$ or $x = 5$

2) Sridharachrya's Method

In this quadratic equation $ax^2 + bx + c = 0$ is solved by using formula

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which gives us $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Q. Solve quadratic equation $x^2 - 2x - 15 = 0$

Solution:

$X1 = \frac{-b+\sqrt{b^2-4ac}}{2a} = 5$

$X2 = \frac{-b-\sqrt{b^2-4ac}}{2a} = -3$

Q. In the following question two equations are given. You have to solve both the equations and find the relation between x and y.

I. $x^2 = 625$

II. $y = \sqrt{625}$

Solution:

We will solve both the equations separately. x^2

= 625

⇒ x = +25 or -25 (we will consider two values of x because of x^2) y =

$\sqrt{625}$

⇒ y = 25 (The square root is used to refer to only the positive square root i.e.
$\{\sqrt{x^2}=|x|\}$.)

∴ $x \leq y$

Q. In the given question, two equations numbered I and II are given. You have to solve both the equations and find the relation between m and n.

I) $m = \sqrt{324}$

II) $n^2 - 16n - 36 = 0$

Solution:

Value of m	Value of n	Result
18	18	m = n
18	-2	m > n

$m = \sqrt{324}$

$\Rightarrow$ m = 18

$n^2 - 16n - 36 = 0$

$\Rightarrow n^2 - 18n + 2n - 36 = 0$

$\Rightarrow n (n - 18) + 2(n - 18) = 0$

$\Rightarrow (n - 18) (n + 2) = 0$

$\Rightarrow$ n = (18, - 2)

Hence, m ≥ n.

Cubic Equation

Polynomial equations with degree 3 i.e., ax3 + bx2 + cx + d = 0 are called as cubic equations. Some examples of cubic equations are as follows –

$x^3 + 2x^2 + 3x + 4 = 0$

$2x^3 + 12x^2 + 30x + 48 = 0$

X= $\sqrt[3]{625}$

Q. In the given question, two equations numbered I and II are given. You have to solve both the equations and mark the appropriate answer.

X= $\sqrt[3]{15625}$

y^2 = 625

Solution:

X= $\sqrt[3]{15625}$ = 25
Y= 625
Y= (+25, -25)
Y ≤ X

AVERAGE

What is the first thing that comes to your mind after hearing average?

In simple words we can say that average is that common value which may be assigned to all and after doing this the end result will be the same.

The average of the number of quantities of observations of the same kind is their sum divided by their number. The average is also called average value or mean value or arithmetic mean.

Average= $\frac{Sum\ of\ Terms}{Number\ of\ Terms}$

- The result obtained by adding several quantities together and then dividing this total by the number of quantities is called Average.
- The main term of average is equal distribution of a value among all which may distribute persons or things. We obtain the average of a number using formulae that is the sum of observations divided by Number of observations.
- Here are Average based on some facts and formulas and some shortcut tricks with examples. Below are some more examples for practicing.

Formula:

- Average = (Sum of observations / Number of observations).

Find the Average Speed

- If a person travels a distance at a speed of x km/hr and the same distance at a speed of y km/hr then the average speed during the whole journey is given by- $\frac{2xy}{x+y}$
- If a person covers A km at x km/hr and B km at y km/hr and C km at z km/hr, then the average speed in covering the whole distance is $\frac{A+B+C}{\frac{A}{X}+\frac{B}{Y}+\frac{C}{2}}$

Note-

- If the average age is increased, Age of new person = Age of separated person + (Increase in average × total number of persons)
- If the average age is decreased, Age of new person = Age of separated person - (Decrease in average × total number of persons)

When a person joins the group- In case of increase in average

- Age of new member = Previous average + (Increase in average × Number of members including new member)

In case of decrease in average

- Age of new member = Previous average - (Decrease in average × Number of members including new member)

In the Arithmetic Progression, there are two cases when the number of terms is odd and the second one is when the number of terms is even. So, when the number of terms is odd the average will be the middle term

- When the number of terms is even then the average will be the average of two middle terms.

Average

An average or an arithmetic mean of given data is the sum of the given observations

divided by number of observations

Important Formulae Related to Average of numbers

1. Average of first n natural number=(n+1)/2
2. Average of first n even number = (n+1)
3. Average of first n odd number = n
4. Average of consecutive number = (First number + Last number)/2
5. Average of 1 to n odd numbers = (Last odd number+1)/2
6. Average of 1 to n even numbers = (Last even number+2)/2
7. Average of squares of first n natural numbers = [(n+1) (2n+1)]/6
8. Average of the cubes of first n natural number =[n(n+1)2]/4
9. Average of n multiples of any number =[Number×(n+1)]/2

Concept 1 If the average of n1 observations is a1; the average of n2 observations is a2 and so on, then Average of all the observations = (n1× a_1+n2 ×a2+......)/(n1+n2+........)

Concept 2 If the average of m observations is a and the average of n observations taken out of is b, then Average of rest of the observations=(ma-n(2)/(m-n)

Example1:

A man bought 20 cows in RS. 200000. If the average cost of 12 cows is Rs. 12500, then what will be the average cost of remaining cows?

Solution:

Here m = 20, n = 12, a = 10000, b = 12500

average cost of remaining cows (20-8) cows = (2010000 -12×12500)/ (20-8) =Rs 6250

Concept 3

If the average of n students in a class is a, where average of passed students is x and average of failed students is y, then

Number of students passed= [Total Students (Total Average-Average of failed students)]/ (Average of passed students-Average of failed students) = [n(a-y)]/(x-y)

Example2:

In a class, there are 75 students whose average marks in the annual examination is 35. If the average marks of passed students is 55and average marks of failed students is 30, then find out the number of students who failed.

Solution:

Here, n = 75, a = 35, x = 55, y = 30

Number of students who passed = 75(35-30)/ (55-30) = 15

Number of students who failed = 75-15 = 60

Concept 4

If the average of total components in a group is a, where average of n components (1st part) is b and average of remaining components (2nd part) is c, then Number of remaining components (2nd part) =[n(a-(2)]/(c-(1)

Example3:

The average salary of the entire staff in an office is Rs. 200 per day. The average salary of officers is Rs. 550 and that of non-officers is Rs. 120. If the number of officers is 16, then find the numbers of non-officers in the office.

Solution:

Here n= 16, a = 200, b = 550, c = 120

Number of non -officer = 16(200-550)/ (120-200) = 70

Average Speed

Average speed is defined as total distance travelled divided by total time taken.

Average speed=Total distance travelled/ Total time taken

Case 1

If a person covers a certain distance at a speed of A km/h and again covers the same distance at a speed of B km/h, then the average speed during the whole journey will be 2AB/A+B

Case 2

If a person covers three equal distances at the speed of A km/h, B km/h and C km/h respectively, then the average speed during the whole Journey will be 3ABC/ (AB+BC+C (1)

Case 3

If distance P is covered with speed x, distance Q is covered with speed y and distance R is covered with speed z, then for the whole journey, Average speed= (P+Q+R+....)/ (P/x+Q/y+R/z+...)

Example 4:

A person covers 20 km distance with a speed of 5 km/h, then he covers the next 15 km with a speed of 3 km/h and the last 10 km is covered by him with a speed of 2 km/h. Find out his average speed for the whole journey.

Solution:

Average speed = (20 +15 +10)/ (20/5+15/3+10/2) = 3(3/14)

Case 4

If a person covers P part of his total distance with speed of x, Q part of total distance with speed of y and R part of total distance with speed of z,then Average speed=1/(P/x+Q/y+R/z+......)

Example 5: The average of 6 consecutive even numbers is 21. Find the largest number?

Solution:

Largest no. = A + (n−1)

A = average

n = no. of terms Largest no.

= 21 + (6 -1) = 26

Example 6: The average of 6 consecutive odd numbers is 22. Find the smallest number?

Solution:

Smallest no. = A - (n - 1)

A = average

n = no. of terms

Smallest no. = 22 - (6 - 1) = 17

Example 7: The average of 5 consecutive even numbers is 46. Find the smallest number?

Solution:

Smallest no. = A - (n - 1)

A = average

n = no. of terms

Smallest no. = 46 -(5 -1) = 42

Example8: Find the average of the first 100 natural numbers?

Solution:

Average = $\frac{(n+1)}{2} = \frac{(100+2)}{2}$ = 50.5

Example 9: The average of 5 numbers is 29. If one number is excluded, the average becomes 27. Find the excluded number?

Solution:

Excluded no. = (5×29 - 4×27)

= (145-108) = 37

Example 10: The average age of 36 students is 15 years. When the teacher's age is included in it, the average increases by 1. What is the teacher's age?

Solution:

Teacher's age = (37×16 - 36×15)

= (592-540) = 52

Example 11: **The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 40 kg. What is the weight of a new person?**

Solution:

Total weight increased = 8 x 2.5 = 20 kg weight of the new person = 40 + 20 = 60 kg

Example 12: The average weight of 10 persons decreases by 2.5 kg when a new person comes in place of one of them weighing 70 kg. What is the weight of a new person?

Solution:

Total weight decreased = 10 x 2.5 = 25 kg Weight of the new person = 70 - 25 = 45 kg

Example 13: A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3 runs. Find his average after the 17th inning.

Solution:

Let the average after 17th inning = X and average after 16th inning = (X - 3)

16(X - 3) + 87 = 17X

16X - 48 +87 = 17X

X = 39

Example 14: The average of 11 results is 60. If the average of first 6 results is 58 and that of last 6 results is 63. Find the 6th result?

Solution:

A11 = 60

Average of first 6 (A6) = 58

Average of last 6 (A6) = 63

6th result = (58 × 6 + 63 × 6 - 60 × 11)

= (348+378) - 660

= 726 -660

= 66

Example 15: **The average of a, 11,23 and 17 is 15 and the average of a, b,12 and 25 is 16. Find the value of a: b?**

Solution: a + 11 + 23 +17 = 15 x 4

a = 9

a + b + 12 + 25 = 16 x 4

a + b = 27

9 + b = 27

b = 18

a: b = 9: 18

= 1: 2

Example 16: The average age of all the 100 employees in an office is 29 years, where 2/5 employees are female and the ratio of average age of male to female is 5: 7. Find the average age of female employees?

Solution:

$60 \times 5x + 40 \times 7x = 29 \times 100$

$300x + 280x = 2900$

$x = 5$

average age of female employees

$= 7x$

$= 7 \times 5$

= 35 years

Example 17: The average of two numbers A & B is 20, an average of B & C is 19 and average of C & A is 21, So find the value of A?

Solution:

$A + B = 40$

$B + C = 40$

$C + A = 42$

On adding above three

$2(A + B + (3) = 40 + 38 + 42 = 120$

$= A + B + C = 60$

$A = (A + B + (3) – (B + (3)$

$= 60 – 38 = 22$

Example 18: Three years ago, the average age of a family of 5 members was 17 years. A baby having been born; the average age of the family is the same today. The present age of the baby is:

Solution:

Total age of 5 members, 3 years ago

= (17×5) = 85 years

Total age of 5 members, now = [85+(3×5)]

= 85 +15 = 100 years

Total age of 6 members now = (17×6)

= 102 years

The age of the baby = (102-100) = 2 years.

Example 19: **The average temperature of a town in the first four days of a month was 58 degrees. The average for the second, third, fourth and fifth days was 60 degrees. The temperature of the first and fifth days was in the ratio 7:8, then what is the temperature on the fifth day?**

Solution:

First four days average Temperature =580

1,2,3, 4th days total temp. = 58×4 = 232

Then 2,3,4,5 days total temp. = 60×4 = 240

Let the unknown temp, be x

5th day – 1st day = 240-232

=8(2,3,4 days temp. is common)

Given the ratio of first and fifth day is 7: 8

8x-7x=8

x=8

Fifth day's temperature = 8x=8x8=64

Practice Questions:

Q1. Shubh was conducting an experiment in which the average of 11 observations came to be 90, while the average of first five observations was 87, and that of the last five was 84. What was the measure of the 6th observation?

(1)165

(2)150

(3)145

(4)135

(5)125

Ans-(4) 135

Q2. A batsman has a certain average of runs for 12 innings. In the 13th innings he scored 96 runs, thereby increasing his average by 5 runs. What is his average after the 13th innings?

(1)64

(2)48

(3)36

(4)72

(5)89

Ans-(3) 36

Q3. There was one mess for 30 boarders in a certain hostel. If the number of boarders was increased by 10, the expenses of the mess increased by Rs. 40 per month, while the average expenditure per head diminished by Rs. 2. Find the original monthly expenses.

(1) Rs. 390

(2) Rs. 360

(3) Rs. 410

(4) Rs. 480

(5) Rs. 450

Ans-(2) Rs 360

Q4. The mean monthly salary paid to 75 workers in a factory is Rs. 5680. The mean salary of 25 of them is Rs. 5400 and that of 30 others is Rs. 5700. The mean salary of remaining workers is:

(1) Rs. 5000

(2) Rs. 7000

(3) Rs. 6000

(4) Rs. 8000

(5) Rs. 9000

Ans-(3) Rs. 6000

Q5. Of the three numbers, the first is twice the second and the second is twice the third. The average of these three numbers is 21. Find the largest number.

(1) 36

(2) 38

(3)47

(4) 48

(5) 35

Ans-(1) 36

Q6. In a mathematics exam, a student scored 30% marks in the first paper out of a total of 180. How much should he score in the second paper out of a total of 150, if he is to get an overall average of 50%?

(1) 74%

(2) 76%

(3)70%

(4) 80%

(5) 75%

Ans-(1) 74%

Q7. The average marks of a student in 8 subjects are 87. Of these, the highest marks are 2 more than the next in value. If these two subjects are eliminated, the average marks of the remaining subjects are 85. What is the highest score?

(1) 91

(2) 94

(3)89

(4) 96

(5) 92

Ans-(2) 94

Q8. An officer's pension on retirement from service is equal to half the average salary during the last 36 months of his service. His salary from 1 January, 2014 is Rs. 3800 per month with increment of Rs. 400 on 1 October 2014, October 2015 and 1 October, 2016. If he retires on 1 January, 2017, what pension does he draw?

(1) Rs. 2100

(2) Rs. 2150

(3) Rs. 2200

(4) Rs. 2250

(5) Rs. 2300

Ans-(2) Rs. 2150

Q9. In a one-day cricket match, Virat, Sehwag, Sachin, Dhoni and Yuvraj scored an average of 39 runs. Dhoni scored 7 more than Yuvraj. Yuvraj scored 9 fewer than Virat. Sehwag scored as many as Dhoni and Yuvraj combined; and Sehwag and Sachin together scored 110 runs between them. How many runs did Sachin score?

(1) 47

(2) 51

(3)53

(4) 49

(5) 57

Ans-(5) 57

Q10. The average of marks obtained by 120 candidates was 35. If the average of the passed candidates was 39 and that of the failed candidates was 15, then the number of those candidates, who passed the examination was:

(1) 100

(2) 110

(3)120

(4) 150

(5) 115

Ans-(1) 100

Q11. A train travels from A to B at the rate of 20kmper hour and from B to A at the rate of 30 km/hr. What is the average rate for the whole journey?

(1) 24 km/hr

(2) 25 km/hr

(3) 26 km/hr

(4) 28 km/hr

(5) None of these

Ans-(1) 24km/hr

Q12. The average salary of the entire staff in an office is Rs 120 per month. The average salary of officers is Rs 460 and that of non- officers is Rs 110. If the number of officers is 15, then find the number of non –officers in the office.

(1) 500

(2) 510

(3) 520

(4) 550

(5) None of these

Ans-(2) 510

Q13. There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs. 42 per day while the average expenditure per head diminishes by Rs1. Find the original expenditure of the mess.

(1) Rs. 400

(2) Rs. 340

(3) Rs. 420

(4) Rs. 450

(5) Rs. 300

Ans-(3) Rs. 420

Q14. The average age of a jury of 5 is 40, if a member aged 35 resigns and a man aged 25 becomes a member, then the average age of the new jury is

(1) 30

(2) 38

(3)40

(4) 42

(5) 36

Ans-(2)38

Q15. The average weight of 8 person is increased by 2.5 kg when one of them whose weight is 56 kg is replaced by a new man. The weight of the new man is:

(1) 58.5 kg

(2) 76 kg

(3)20 kg

(4) 64 kg

(5) None of these

Ans-(2)76kg

TIME AND WORK

Concepts

1. If A can do a piece of work in 10 days, then in 1 day, A will do $\frac{1}{10}$ $part\ of\ the\ total\ work.$
2. If A is thrice as good as B, then
a) In a given amount of time, A will be able to do 3 times the work B does. Ratio of work done by A and B (in the same time) = 3: 1.
b) For the same amount of work, B will take thrice the time as much as A takes. Ratio of time taken by A and B (same work done) = 1: 3.
3)Efficiency is directly proportional to the work done and inversely proportional to the time taken.

Basic Questions

Q1. A does a work in 10 days and B does the same work in 15 days. In how many days will they do the same work together?

Solution:
A does a work in 10 days
A□s 1 day□s work= $\frac{1}{10}$

B does a work in 15 days

B□s 1 day's work= $\frac{1}{15}$

Adding equation 1 and 2, we get,

A and B□s 1day□s work=$\frac{1}{10} + \frac{1}{10} = \frac{1}{6}$

∴ A and B will together take 6 days to do the work.

Q2. A alone can do a job in 40 days. In how many days can B alone do the job, if together they can do the job in 8 days?

Solution:
⇒ Efficiency of A: Efficiency of A + B = (1/40) : (1/8) = 1 : 5

⇒ Efficiency of B / Efficiency of A = (5 - 1)/1 = 4/1 B is

4 times efficient than A,

⇒ Number of days taken by B = 1/4 × Number of

days taken by A = 40/4 = 10 days

When three persons do a work

Q3. A and B can do a work in 3 days; B & C can do it in 4 days and A & C can do it in 6 days. In how many days will A, B & C finish it, if they work together?

Solution:

⇒ Work done by A & B in 1 day =1/3 ...(I)

⇒ Work done by B & C in 1 day =1/4 ... (II)

⇒ Work done by A & C in 1 day =1/6

... (III)

Adding I), II) & III)

⇒ 2 × (work done by A, B & C in 1 day) = 1/3 + 1/4 + 1/6

∴ Work done by A, B & C in 1 day together =1/2 (1/3 + 1/4 + 1/6)

= 1/2((4 + 3 + 2) /12) = 3/8

∴ No. of days taken to complete the job together = 8/3 days

Q4. A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would A take separately to do the same work?

Solution:

One day work of A and B = 1/12

One day work of B and C = 1/15

One day work of C and A = 1/20
One day work of (A + B), (B + C) and (C + A) = 1/12 + 1/15 + 1/20

= 12/60 = 1/5

One day work of (A + B + C) = (1/2) × (1/5) = 1/10

One day work of A = One day work of (A + B + C) - One day work of (B + C)

= 1/10 – 1/15 = 2/60 = 1/30

∴ A takes 30 days alone to do the same work.

Questions Based on Efficiency

Q5. Anil is thrice as good a workman as Arun. Together they can do a job in 12 days. In how many days will Arun finish the work alone?

Solution:

Anil is thrice as good a workman as Arun

If Anil can finish a work in x days Arun can finish the same work in 3x days.

∴ 1 day's work by Anil = 1/x

1 day's work by Arun = 1/3x

Together they can do a job in 12 days

1 day's work by (Anil + Arun) = $\frac{1}{X} + \frac{1}{3X} = \frac{4}{3X}$

Work done in 15 days = (4/3x) × 12 = (16/x)

If Arun has to finish that work alone, time taken by Arun to finish the work alone

$$=\frac{\frac{16}{x}}{\frac{1}{3x}} = 48 days$$

Q6. P is twice as good as Q and together they finish a piece of work in 36 days. The number of days taken by P alone to finish the work?

Solution:

Given,

P is twice as good as Q.

⇒ (P's 1 day's work) / (Q's 1 day's work) = 2 / 1

Given,

⇒ (P + Q)'s 1 day's work = 1/36

⇒ P's 1 day's work = (1/36) × (2/3) = 1/54

∴ P alone can finish work in 54 days.

Work and wages

Concepts

Ratio of wages of persons doing a work is directly proportional to the ratio of efficiency of the persons

Q7. A and B can complete a piece of work in 15 days and 10 days respectively. They got a contract to complete the work for Rs. 75000. The share of B (in Rs.) in the contracted money will be:

Ratio of number of days taken by A and B to complete the work = 15: 10 = 3: 2

∴ Ratio of efficiency of A and B = 2: 3

Let their share is in the ratio of 2x and 3x

Now,

2x + 3x = 75000

⇒ 5x = 75000

∴ x = 15000

∴ Share of B = 3x = 15000 × 3 = RS. 45000

When two or more persons do a work on alternate days or hours

Q8. A and B can complete a piece of work in 10 days and 15 days respectively when working alone. Starting with A, they work on alternate days. In how many days will the work be completed?

A. Given,

⇒ A's 1 day's work = 1/10

⇒ B's 1 day's work = 1/15

(A + B)'s 2 days' work = 1/10 + 1/15 = 1/6

To complete work (A + B) 12 days' work = 6 × 1/6 = 1

Total time taken by both if they work alternatively = 12 days.

MHDE/W based

Concept:

More men, less days : $M \propto \frac{1}{D}$

More men, less hours : $M \propto \frac{1}{H}$

MDH = Constant

More efficiency, less days: $E \propto \frac{1}{D}$

More men, more work : $M \propto W$

$\frac{M\,D\,H\,E}{W}$ = Constant

Where M = Men, D = Days, H = Hours, E = Efficiency, W = Work

Q9. 15 men can complete a task in 8 days. Three days after they started the work, 3 more men joined. In how many days will all of them together complete the remaining work?

Total work done by 15 men in 8 days = 15 × 8 = 120

Work done by 15 men in 3 days = 15 × 3 = 45

Remaining work after 3 days = 120 – 45 = 75

Days to complete remaining work by 18 men = 75/18 =

25/6 = $4\frac{1}{6} Days$

Food based

This type of questions based on quantity of food required for feeding given number of persons or number of days for which given quantity of food lasts

Q10. 1200 soldiers in a fort had enough food for 28 days. After 4 days, some soldiers left the fort. Thus, food lasted for 32 more days. How many soldiers left?

Solution:

Let the number of soldiers who left the fort be x. As
we know,
$M_1 \times D_1 = M_2 \times D_2 + M_3 \times D_3$
Given, M_1 = 1200 soldiers, D_1 = 28 days, M_2 = 1200 soldiers, D_2 = 4 days, M_3 = (1200– x) soldiers and D_3 = 32
$M_1 \times D_1 = M_2 \times D_2 + M_3 \times D_3$
1200 × 28 = 1200 × 4 + (1200 – x) × 32
(1200 × 28) – (1200 × 4) = (1200 – x) × 32
1200 (28 – 4) = (1200 – x) × 32
(1200 × 24) /32 = (1200 – x)
x = 300

Man, Woman and Boys
When men, women and Boys together or alone do work.

Q11. It takes the same time of 66 days for 12 men to finish the same work as it takes for 24 boys. If we assign this work to 20 men and 10 boys, in how many days will they be able to finish this work?

Solution:

Let us assume the total work to be W.
Let the efficiency of each man be M units/day, and that of each boy be B units/day.

⇒ W = 12 × M × 66 = 24 × B × 66

∵ 12 × M × 66 = 24 × B × 66,

(M/B) = 2

⇒ M = 2B
The task is to get 20 men and 10 boys to do the same. Let the no. of days be D.

⇒ W = (20M + 10B) × D
Replacing all the variables in terms of B, we have 24

× B × 66 = (20 × 2B + 10B) × D⇒ 24 × B × 66 = 50 × B ×

D

D= $31\frac{17}{25} Days$

∴ Time taken by 20 men and 10 boys to finish the same work =$31\frac{17}{25} Days$

PROFIT AND LOSS

Profit and loss is the second pillar of math out of four (namely percentage, profit and loss, ratio and proportion and average). In pre-exams 2-3 Word problems are asked directly and in the DI section many times DI based on profit and loss are asked. (like income expenditure DI sets)

If we talk about the mains exam the same pattern is followed in word problem section and in data interpretation section its importance becomes more as some direct DI sets are asked in examinations nowadays especially in PO mains exams. In short, we can say that this is a section you cannot leave for exams.

Profit and Loss terminologies

1. Cost Price (CP):
The price, at which an article is purchased, is called its cost price, usually denoted by C.P. In simple words we can say the money that goes out of the pocket of the seller is added to its cost price. (like the price paid by the seller to the wholesaler, transportation cost, labour charges and different types of miscellaneous charges) Sometimes it is denoted by expenditure.

2. Selling Price (SP):
The price, at which an article is sold, is called its selling prices, abbreviated as S.P. In short, we can say that the amount that comes in the pocket of the seller is added to its selling price. Sometimes it is denoted by income

3. Marked Price:
When we purchase any item or article, we see that a price is marked on it and we pay the same or ask for some discount, this price marked on it is known as marked price. Sometimes it is denoted by labelled price or first price.

4. Profit or Gain:
If S.P. is greater than C.P., the seller is said to have a profit or gain. or we can say if the seller got somewhere more what he spends then its a case of profit.

5. Loss:
If S.P. is less than C.P., the seller is said to have incurred a loss. or we can say if the seller got somewhere less what he spends then its case of loss.

6. Profit percentage = Profit percentage is sometimes calculated on CP and sometimes SP. If profit percent is calculated on CP, then

P%= $\frac{SP-CP}{CP}\times 100$

If profit percent is calculated on SP, then

P%= $\frac{SP-CP}{SP}\times 100$

7. Loss percentage=Loss percentage is sometimes calculated on CP and sometimes SP. If loss percent is calculated on CP, then

P%= $\frac{CP-SP}{CP}\times100\%$

If loss percent is calculated on SP, then

P%= $\frac{CP-SP}{SP}\times100\%$

8. Markup price=Usually sellers mark the price of any article more than its cost price this percentage is called markup price. a percentage is sometimes calculated on CP and sometimes SP.

Markup price= $\frac{MP-CP}{CP}$×100%

9. Discount percentage=Usually sellers give any article at some lower price than what is written on it. This percentage decrease in price of an article is called discount.
Discount percent is always calculated on marked price

D%= $\frac{MP-SP}{MP}\times100$

Note-Sometimes discount is given by the seller but not directly but in some conditional form like if you will buy 5 articles, I will give you 1 article absolutely free this is also a case of discount. How to calculate discount percent in these cases we will study ahead.

10. Equivalent discount percent= When two successive discounts are given then equivalent discount may be calculated easily by this formula

Equivalent discount=X+Y $-\frac{XY}{100}$

Points to Remember

- CP +Profit=SP
- CP-Profit=SP
- SP+ Discount=M P
- If there is a PROFIT of x%, the calculating figures would be 100 and (100 + x).
- If there is a PROFIT of x%, the calculating figures would be 100 and (100 + x).
- Calculating figures be Cost Price and Selling Price respectively.

Now let's discuss some examples and then we will study some important formulae

Q1. A shopkeeper fixes the marked price of an item 35% above its cost price. The percentage of discount allowed to gain 8% is?
(1) 18%
(2) 20%
(C)22%
(4) 24%

(5) None of these

Answer: (2) 20%
Explanation:
Let the cost price = Rs.100/-
then, Marked price = Rs.135/-
Required gain = 8%,
So, Selling price = Rs.108/-
Discount = 135 -108 = 27
Discount% = $\frac{27}{135} \times 100\%$
=20%

Q2. A person incurs a loss of 5% by selling a watch for Rs. 1140. At what price should the watch be sold to earn 5% profit?
(1) Rs.1200
(2) Rs.1230
(3) Rs.1260
(4) Rs.1290
(5) None of these

Answer: (3) Rs.1260
Explanation:
Let CP=100
SP=95
New SP=105
Required answer= $\frac{1140}{95} \times 105 = 1260$

Q3. If the cost price of 12 bananas is equal to the selling price of 8 bananas, the gain percent is?
(1)12%
(2)50%
(3)30%
(4)60%
(5) None of these
Answer: 2
Explanation:
We know we will need to gain an amount to get gain percent, right. So, let's get gain first.
Let the cost price of 1 banana is Rs 1
Cost of 8 bananas = Rs 8
Selling price of 8 bananas= 12
Gain = 12 -8 = 4
Gain%= $\frac{4}{8} \times 100\%$ =50%

Some Important Concept

1. If a person sells two similar articles, one at a gain of a% and another at a loss of a%, then the seller always has a loss which is given by

Loss%= $(\frac{a}{10})^2$

don't afraid this formula came from successive concepts and you can easily calculate it.

2. If a'th part of some items is sold at x% loss, then required gain per cent in selling rest of the items in order that there is neither gain nor loss in whole transaction, is

$$\frac{ax}{(1-a)}\%$$

3. If cost price of 'a' articles is equal to the selling price of 'b' articles, then profit percentage can be directly calculated by

$$\frac{(a-b)}{b}\times 100\%$$

4. If a dishonest trader professes to sell his items at CP but uses false weight, then

Gain%= $\frac{E}{T}\times 100\%$

Where E= error

T= true value

5. If 'a' part of an article is sold at x% profit/loss, 'b' part at y% profit/loss and c part at z% profit/loss and finally there is a profit/loss of Rs. R, then Cost price of entire article

=$\frac{R}{ax+by+cz}\times 100$

Some Formulae:

1) If an article is sold at a profit/gain of 30%, then S.P. = 130% of the C.P.

2) If an article is sold at a loss of 20%, then S.P. = 80% of the C.P.

3) When there are two successive Profit of x % and y % then the resultant profit per cent is given by

   X+Y + $\frac{XY}{100}$

4) If there is a Profit of x% and loss of y % in a transaction, then the resultant profit or loss% is given by

   X-Y-$\frac{xy}{100}$

   Note

   For profit use sign + in previous formula and for loss use – sign.

   if resultant come + then there will be overall profit. if it comes – then there will be overall loss.

5) A man purchases a certain no. of articles at m a rupee and the same no. at n a rupee. He mixes them together and sold them at p a rupee then his gain or loss %

$(\frac{2mn}{(m+n)} - 1)\times 100$

Note + =Profit, - = Loss

6) If a seller marks his goods at x% above his cost price and allows purchasers a discount of y % for cash, then overall gain or loss

X-Y- $\frac{xy}{100}$

Profit or loss according to sign + = Gain, - =Loss.

Solved Examples

Type 1:

The cost price of 40 articles is the same as the selling price of 25 articles. Find the gain per cent.

(a) 65%

(b) 60%

(c) 15%

(d) 75%

Answer: (b) Gain per cent $\frac{40-25}{25}\times 100 = 60\%$

Type 2:

Bananas are bought at the rate of 6 for Rs. 5 and sold at the rate of 5 for Rs. 6. Profit per cent is

(a) 36%

(b) 42%

(c) 44%

(d) 48%

Answer: (c)

To avoid fraction, let the number of bananas bought LCM of 5 and 6 = 30

CP of 30 bananas = 5×5 = Rs. 25

SP of 30 Bananas = 6× 6

= Rs. 36

Profit = Rs. (36-25) = Rs. 11

Profit %= $\frac{11}{25}\times 100 = 44\%$

Type 3:

A man bought oranges at the rate of 8 for Rs 34 and sold them at the rate of 12 for Rs. 57. How many oranges should be sold to earn a net profit of Rs 45?

(a) 90

(b) 100

(c) 135

(d) 150

Answers: (a) 90

Let the man buy 24 (LCM of 8 and 12) oranges.

C.P. of 24 oranges = $\frac{34}{8}\times 24$=102

S.P. of 24 oranges = $\frac{27}{12}\times 24$ = 114

Gain = 114 – 102 = Rs.12

Rs. 12 = 24 oranges

Rs. 45 = $\frac{24}{12}\times 45$ = 90 oranges

Type 4:

A shopkeeper earns a profit of 12% on selling a book at 10% discount on printed price. The ratio of the cost price to printed price of the book is ?

(a) 45 : 56

(b) 50 : 61

(c) 90 : 97

(d) 99 : 125

Answer: (a) 45 : 56

C.P. of the book = Rs. x

Printed price = Rs. Y

$$\frac{y\times 90}{100} = \frac{X\times 112}{100}$$
$$\frac{x}{y} = \frac{45}{56}$$

Type 5:

A dealer sold two types of goods for Rs 10,000 each. On one of them, he lost 20% and on the other he gained 20%. His gain or loss per cent in the entire transaction was

(a) 2% loss

(b) 2% gain

(c) 4% gain

(d) 4% loss

Answers: (d) 4%loss

Here, S.P. is the same, hence there is always a loss. Loss per cent = $\frac{20\times 20}{100}$ × =4%

Type 6:

On selling an article for Rs170, a shopkeeper loses 15%. In order to gain 20%, he must sell that article at rupees:

(a) 215.50

(b) 212.50

(c) 240

(d) 210

Answer: (c) 240

C.P. of article = $\frac{170}{85}\times 120$ =240

Type 7:

An article is sold at a loss of 10%. Had it been sold for Rs. 9 more, there would have been a gain of 12.5% on it. The cost price of the article is

(a) Rs. 40

(b) Rs. 45

(c) Rs. 50

(d) Rs. 35

Answers:(a) Rs. 40

	CP	SP

Before	100	90
After	100	112.5

Now difference of S.P 22.5% = 9
So required answer= 40

Type 8:
A sell a suitcase to B at 10% profit. B sells it to C at 30% profit. If C pays Rs 2860 for it, then the price at which a bought it is
(a) 1000
(b) 1600
(c) 2000
(d) 2500
Answer: (c) 2000
If the C.P. of the suitcase for A be Rs. x, then
X $\times \frac{110}{100} \times \frac{130}{100} = 2860$

x= Rs. 2000

SIMPLIFICATION

Helping Hands:

1. Digits - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

2. Types of numbers.

(i) Natural numbers. = {1, 2, 3, 4, ...}

(ii) Whole numbers = {0, 1, 2, 3, 4..........}

(iii) Integers = {......, -3, -2, -1, 0, 1, 2, 3, ...}

(iv) Real numbers = {----, 2.8, -2, -10, 1, 1.9, --2, 3, 3.12, 3.13-----}

(v) Even numbers = {2, 4, 6, ---------}

(vi) Odd numbers = {1, 3, 5, 7-----------}

(vii) Prime numbers = {2, 3, 5, 7, 11, 13, 17, 19, - -----}

3.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a+b).(a-b)$

$(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)$

$a^3 + b^3 = (a+b)\,(a^2 + b^2 - ab)$

$a^3 - b^3 = (a-b)\,(a^2 + b^2 + ab)$

ADDITION & SUBTRACTION

Ex-1. ? = 8+88+888+8888+88888

? = 8(1+11+111+1111+11111)

? = 8(12345) ⇒ 98760

Ex-2. ? = 0.8 + 0.88 + 0.888 + 0.8888 + 0.88888

? = 8(0.1+0.11+0.111+0.1111+0.11111)

? = 8(0.54321)

⇒ 4.34568

Ex-3. 8.8 + 8.88 + 8.888 + 8.8888 + 8.88888 =?

First, we can calculate decimal number and then whole no.

From Ex-2.

8(0.54321) = 4.34568

and 8+8+8+8+8 i.e., 8×5=40

Therefore, 40+4.34568 = 44.34568

Ex-4. 8456+3891+4560 =?

= 16907

Ex-5. 3.981+14.34+12.5=?

First, we can balance the number of decimal digits and then use the elimination method.

i.e., 3.981 +14.340 + 12.500 =?

= 30.821

MULTIPLICATION

Some Special Types

1. When the sum of the unit digit is 10 and the remaining digit is same.

Example- 43 × 47

= 4 × (4+1) / 3×7

= 4 × 5 / 21

= 20 / 21

Ans = 2021.

2. When sum of tens digit is 10 and unit digit is same

Example- 46 × 66

= (4 × 6) + 6 / 6 × 6

= 24 + 6 / 36

= 30 / 36

Ans = 3036

3. When the unit digit is 5 in both the numbers and difference between each number is 10.

Example- 75 × 65

= 6 × (7 + 1) / 75

= 48 / 75

Ans = 4875

SQUARE AND SQUARE ROOTS

Learn Square of 1 to 50

Square of 1-50 numbers

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$.................................$50^2 = 2500$

Type-I. Formula Method We know that

$(a + b)^2 = a^2 + 2ab + b^2$ i.e. $(a/b)^2 = a^2 / 2ab / b^2$

Ex-1. $(56)^2$

$= (5/6)^2$

$= 5^2 / 2 \times 5 \times 6 / 6^2$

6 3

= 25 / 60 / 36

= 31 / 3 / 6

= 3136

We break number in two parts i.e., 5 & 6 and follow the rule of $(a+b)^2 = a^2/2ab/b^2$

CUBE & CUBE ROOT

Learn cubes from 1 to 25.

FRACTIONS

Fraction is known as a fraction in which a is called numerator and b is called denominator.

Types of Fractions:

I. Proper Fraction: If the numerator part of a fraction is less than the denominator then the fraction is called proper fraction and proper fraction is always less than 1.

II. Improper fraction: If the numerator of a fraction is greater than denominator then the fraction is called improper fraction. Improper fraction is always greater than 1.

$\frac{5}{4}, \frac{3}{2}, \frac{7}{5}, \frac{11}{8}$............ etc.

III. Mixed Fraction: Mixed with proper fraction: When a proper fraction is mixed with a whole number known as mixed with proper fraction.

e.g., $8\frac{7}{2}$

VBODMAS RULE

V – Vinculum means bar as (-)

B – Bracket- () {} and then [ ]

O – of

D – Division [÷]

M – Multiplication [×]

A – Addition [+]

S – Subtraction [-]

The word 'VBODMAS' represents the order of calculation i.e., order of signs

B	O	D	M	A	S
Brackets	Orders	Divide	Multiply	Add	Subtract

Example 1: 35 ÷ 7 × 5 =?

Solution:

According to the order of VBODMAS, first we solve division and then multiplication

i.e., 35 ÷ 7 × 5 =?

5 × 5 =?

? = 25

Example 2: 35 ÷ 5 of 7 =?

Solution:

According to the order of VBODMAS, first we solve ‘of’ and then division.

i.e., 35 ÷ 5 of 7 =?

35 ÷ 35 =?

? = 1

Example 3: 48 ÷ 12 of 2 + [3 + 17 × 2] =?

Solution:

48 ÷ 24 + 37 =?

2 + 37 =?

? = 39

Example 4: 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 =?

Solution:

2/2 × 2 × 2 × 2 × 2 = ?

? = 1/16